Supplementary material for the paper

Model-based SIR for dimension reduction

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Abstract

This document can be viewed with any pdf reader. However, the animation in Figures 6 and 7 can only be viewed with a recent version of Adobe Reader.

3. Simulation studies

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Table 1: Simulation results for *model 1*: average distance $\Delta(\hat{\boldsymbol{B}}, \boldsymbol{B})$ with the corresponding standard deviation within parenthesis, based on 500 simulations for different sample sizes (n), number of predictors (p) and error standard deviation (σ).

σ	p	n	SIR	SAVE	PHD	DR	MSIR
0.1	5	100	0.8705 (0.17)	0.2159 (0.09)	0.2179 (0.08)	0.2529 (0.10)	**0.1623** (0.13)
		200	0.8824 (0.16)	0.1365 (0.05)	0.1606 (0.06)	0.1321 (0.05)	**0.0735** (0.03)
		500	0.8748 (0.17)	0.0825 (0.03)	0.1004 (0.04)	0.1094 (0.04)	**0.0338** (0.01)
		1000	0.8801 (0.16)	0.0570 (0.02)	0.0742 (0.03)	0.0561 (0.02)	**0.0196** (0.01)
	10	100	0.9161 (0.13)	0.4970 (0.19)	**0.3583** (0.10)	0.4573 (0.16)	0.4477 (0.33)
		200	0.9278 (0.11)	0.2665 (0.07)	0.2582 (0.07)	0.2984 (0.08)	**0.1843** (0.09)
		500	0.9365 (0.10)	0.1451 (0.04)	0.1628 (0.04)	0.1743 (0.05)	**0.0971** (0.03)
		1000	0.9290 (0.12)	0.0986 (0.02)	0.1135 (0.03)	0.1172 (0.03)	**0.0484** (0.01)
	20	100	0.9562 (0.07)	0.8524 (0.14)	**0.5812** (0.11)	0.9083 (0.11)	0.9333 (0.15)
		200	0.9591 (0.07)	0.5624 (0.15)	**0.3974** (0.08)	0.5407 (0.13)	0.5313 (0.34)
		500	0.9685 (0.05)	0.2545 (0.05)	0.2457 (0.05)	0.2523 (0.05)	**0.1759** (0.03)
		1000	0.9614 (0.07)	0.1659 (0.03)	0.1736 (0.03)	0.1876 (0.03)	**0.1335** (0.02)
0.5	5	100	0.8633 (0.18)	0.3026 (0.13)	**0.2434** (0.09)	0.3109 (0.12)	0.3956 (0.30)
		200	0.8778 (0.17)	0.1884 (0.07)	0.1761 (0.06)	0.1715 (0.06)	**0.1692** (0.09)
		500	0.8817 (0.15)	0.1088 (0.04)	0.1083 (0.04)	0.1393 (0.06)	**0.1016** (0.04)
		1000	0.8770 (0.16)	0.0751 (0.03)	0.0766 (0.03)	0.0754 (0.03)	**0.0675** (0.03)
	10	100	0.9258 (0.11)	0.5976 (0.18)	**0.3941** (0.11)	0.6196 (0.20)	0.7556 (0.29)
		200	0.9400 (0.09)	0.3361 (0.10)	**0.2730** (0.07)	0.3584 (0.10)	0.3774 (0.30)
		500	0.9360 (0.11)	0.1811 (0.05)	0.1700 (0.05)	0.2204 (0.06)	**0.1480** (0.04)
		1000	0.9407 (0.09)	0.1259 (0.03)	0.1212 (0.03)	0.1465 (0.04)	**0.1113** (0.03)
	20	100	0.9669 (0.05)	0.8792 (0.12)	**0.6041** (0.11)	0.9290 (0.09)	0.9584 (0.09)
		200	0.9623 (0.07)	0.6641 (0.16)	**0.4295** (0.09)	0.6718 (0.16)	0.8552 (0.25)
		500	0.9656 (0.06)	0.3222 (0.06)	0.2603 (0.05)	0.3188 (0.05)	**0.2236** (0.09)
		1000	0.9676 (0.06)	0.2065 (0.03)	0.1855 (0.03)	0.2345 (0.04)	**0.1540** (0.03)
1.0	5	100	0.8917 (0.13)	0.4583 (0.22)	**0.3054** (0.14)	0.4371 (0.18)	0.6757 (0.30)
		200	0.8932 (0.14)	0.2613 (0.10)	**0.2190** (0.08)	0.2638 (0.10)	0.4654 (0.33)
		500	0.8803 (0.15)	0.1496 (0.06)	**0.1326** (0.05)	0.1938 (0.08)	0.1703 (0.12)
		1000	0.8802 (0.16)	0.1036 (0.04)	**0.0941** (0.03)	0.1039 (0.04)	0.1122 (0.04)
	10	100	0.9357 (0.09)	0.7433 (0.19)	**0.4817** (0.14)	0.7963 (0.19)	0.8957 (0.17)
		200	0.9332 (0.11)	0.4836 (0.16)	**0.3298** (0.08)	0.4693 (0.12)	0.7877 (0.28)
		500	0.9449 (0.08)	0.2526 (0.06)	**0.2098** (0.05)	0.2932 (0.08)	0.2868 (0.21)
		1000	0.9385 (0.09)	0.1716 (0.04)	**0.1467** (0.03)	0.2065 (0.05)	0.1641 (0.04)
	20	100	0.9680 (0.05)	0.9243 (0.09)	**0.7078** (0.13)	0.9516 (0.07)	0.9731 (0.05)
		200	0.9687 (0.05)	0.8028 (0.15)	**0.5079** (0.10)	0.8287 (0.14)	0.9689 (0.07)
		500	0.9677 (0.05)	0.4462 (0.10)	**0.3049** (0.06)	0.4433 (0.08)	0.7715 (0.31)
		1000	0.9689 (0.05)	0.2809 (0.05)	**0.2200** (0.04)	0.3205 (0.06)	0.2528 (0.12)

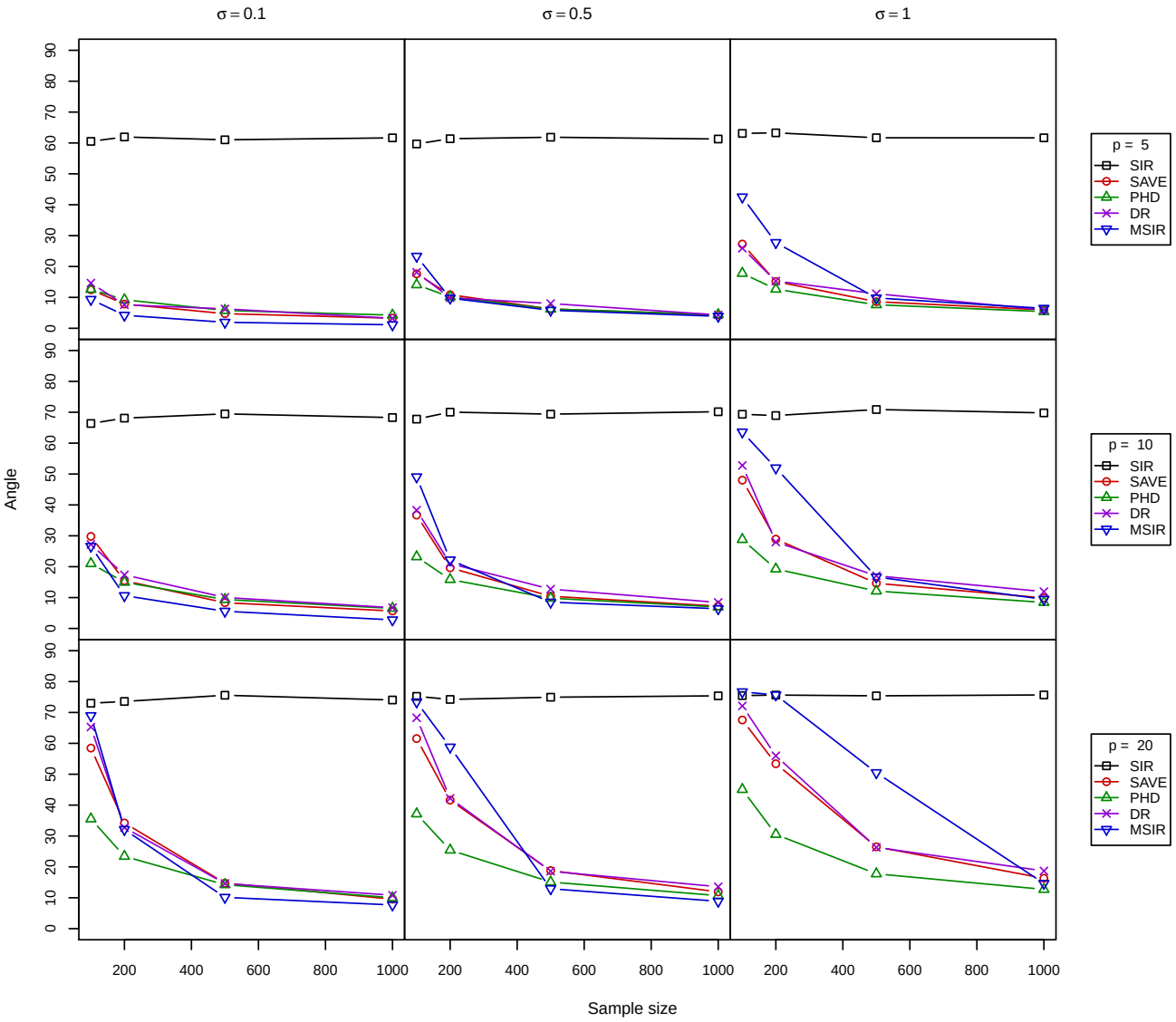

Figure 1: Graphs of simulation results for *model 1*.

Table 2: Simulation results for *model 2*: average distance $\Delta(\hat{\boldsymbol{B}}, \boldsymbol{B})$ with the corresponding standard deviation within parenthesis, based on 500 simulations for different sample sizes (n), number of predictors (p) and error standard deviation (σ).

σ	p	n	SIR	SAVE	PHD	DR	MSIR
0.1	5	100	0.8492 (0.19)	0.5085 (0.26)	0.7883 (0.19)	**0.3151** (0.11)	0.3561 (0.25)
		200	0.8505 (0.19)	0.2085 (0.08)	0.7862 (0.21)	0.1834 (0.06)	**0.1539** (0.09)
		500	0.8306 (0.19)	0.1098 (0.04)	0.7784 (0.20)	0.1405 (0.05)	**0.0725** (0.02)
		1000	0.8446 (0.19)	0.0740 (0.02)	0.8181 (0.19)	0.0769 (0.02)	**0.0509** (0.02)
	10	100	0.9242 (0.11)	0.9687 (0.05)	0.9043 (0.10)	**0.6417** (0.18)	0.7446 (0.28)
		200	0.9248 (0.11)	0.9033 (0.13)	0.8950 (0.11)	0.3738 (0.09)	**0.3582** (0.25)
		500	0.9285 (0.11)	0.2386 (0.07)	0.8930 (0.11)	0.2193 (0.05)	**0.1584** (0.03)
		1000	0.9356 (0.09)	0.1333 (0.03)	0.8979 (0.11)	0.1479 (0.03)	**0.0697** (0.02)
	20	100	0.9655 (0.05)	0.9896 (0.01)	0.9569 (0.05)	**0.9402** (0.08)	0.9616 (0.07)
		200	0.9536 (0.07)	0.9906 (0.01)	0.9597 (0.05)	**0.6894** (0.15)	0.8162 (0.26)
		500	0.9626 (0.06)	0.9870 (0.02)	0.9532 (0.06)	0.3238 (0.05)	**0.2388** (0.08)
		1000	0.9634 (0.06)	0.5984 (0.18)	0.9495 (0.06)	0.2332 (0.04)	**0.1649** (0.02)
0.5	5	100	0.8365 (0.19)	0.6558 (0.26)	0.8064 (0.19)	**0.3597** (0.14)	0.4283 (0.28)
		200	0.8473 (0.19)	0.3052 (0.15)	0.8055 (0.20)	0.2040 (0.07)	**0.1967** (0.12)
		500	0.8423 (0.18)	0.1291 (0.04)	0.7795 (0.21)	0.1500 (0.06)	**0.0912** (0.03)
		1000	0.8566 (0.17)	0.0878 (0.03)	0.7962 (0.19)	0.0878 (0.03)	**0.0537** (0.02)
	10	100	0.9175 (0.12)	0.9719 (0.04)	0.9128 (0.10)	**0.6815** (0.18)	0.8022 (0.24)
		200	0.9362 (0.10)	0.9529 (0.07)	0.9000 (0.11)	**0.3984** (0.09)	0.4349 (0.28)
		500	0.9269 (0.11)	0.4187 (0.17)	0.9083 (0.10)	0.2376 (0.05)	**0.1767** (0.03)
		1000	0.9261 (0.11)	0.1701 (0.04)	0.9020 (0.11)	0.1584 (0.03)	**0.1146** (0.03)
	20	100	0.9622 (0.06)	0.9890 (0.02)	0.9602 (0.05)	**0.9490** (0.06)	0.9678 (0.06)
		200	0.9631 (0.06)	0.9913 (0.01)	0.9564 (0.05)	**0.7195** (0.15)	0.8741 (0.21)
		500	0.9596 (0.07)	0.9871 (0.02)	0.9444 (0.07)	0.3484 (0.05)	**0.2840** (0.15)
		1000	0.9666 (0.06)	0.9108 (0.10)	0.9472 (0.06)	0.2502 (0.04)	**0.1782** (0.02)
1	5	100	0.8394 (0.19)	0.8332 (0.18)	0.8044 (0.19)	**0.4854** (0.19)	0.5931 (0.29)
		200	0.8599 (0.18)	0.6254 (0.26)	0.8013 (0.19)	**0.2917** (0.11)	0.2997 (0.20)
		500	0.8344 (0.19)	0.2535 (0.12)	0.8126 (0.19)	0.1931 (0.07)	**0.1384** (0.05)
		1000	0.8562 (0.17)	0.1326 (0.05)	0.8220 (0.18)	0.1115 (0.04)	**0.0860** (0.03)
	10	100	0.9322 (0.09)	0.9753 (0.03)	0.9108 (0.10)	**0.8049** (0.16)	0.8432 (0.20)
		200	0.9267 (0.11)	0.9670 (0.05)	0.9085 (0.10)	**0.5372** (0.15)	0.5909 (0.30)
		500	0.9332 (0.10)	0.8674 (0.14)	0.9152 (0.09)	0.3144 (0.07)	**0.2258** (0.09)
		1000	0.9293 (0.11)	0.4150 (0.16)	0.9088 (0.10)	0.2069 (0.04)	**0.1528** (0.03)
	20	100	0.9701 (0.05)	0.9891 (0.02)	0.9662 (0.04)	**0.9598** (0.05)	0.9706 (0.05)
		200	0.9662 (0.05)	0.9905 (0.01)	0.9573 (0.05)	**0.8158** (0.13)	0.9444 (0.12)
		500	0.9689 (0.06)	0.9908 (0.01)	0.9549 (0.06)	0.4504 (0.08)	**0.4328** (0.26)
		1000	0.9665 (0.05)	0.9824 (0.02)	0.9604 (0.05)	0.3203 (0.05)	**0.2167** (0.03)

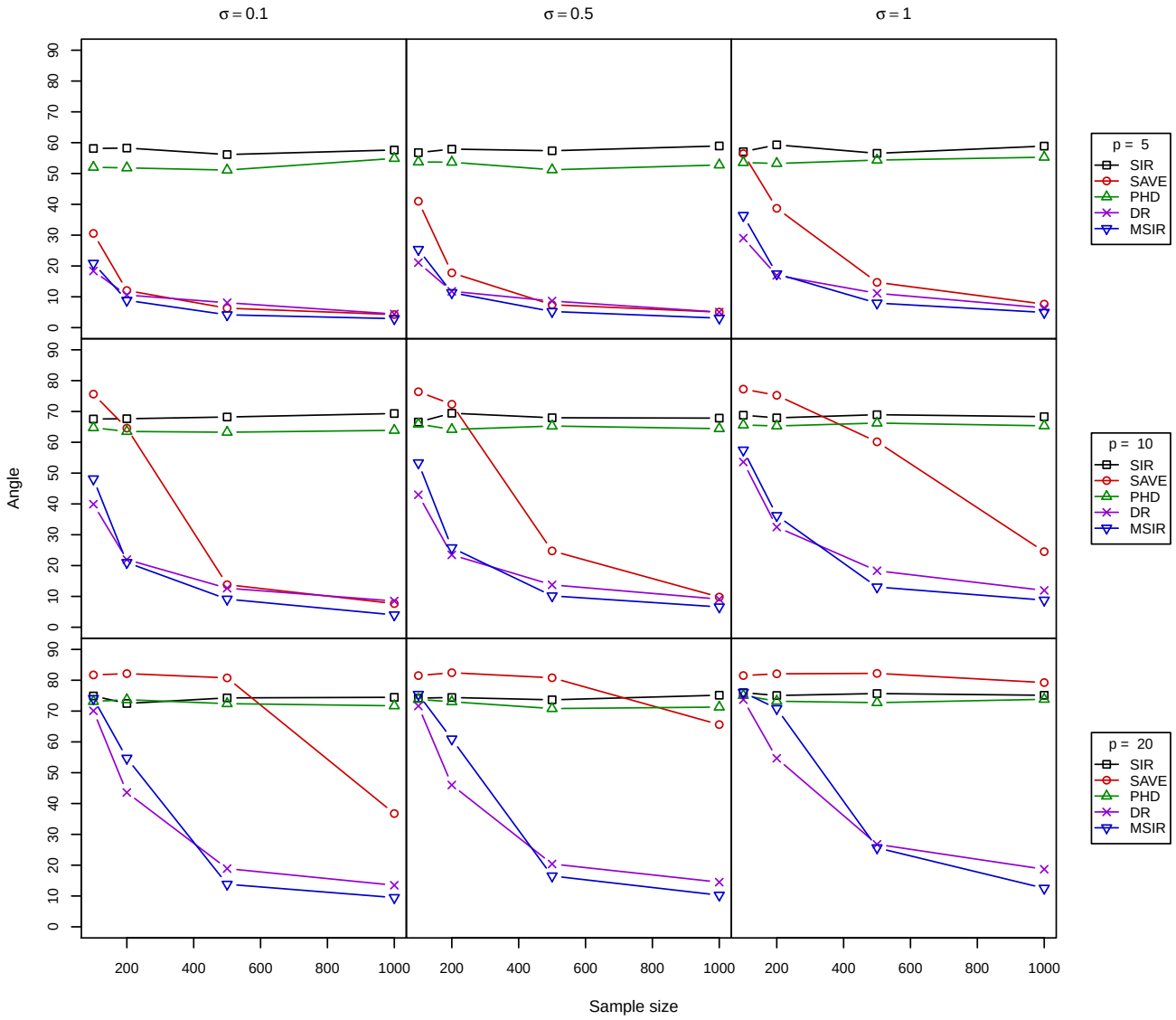

Figure 2: Graphs of simulation results for *model 2*.

Table 3: Simulation results for *model 3*: average distance $\Delta(\hat{\boldsymbol{B}}, \boldsymbol{B})$ with the corresponding standard deviation within parenthesis, based on 500 simulations for different sample sizes (n), number of predictors (p) and error standard deviation (σ).

σ	p	n	SIR	SAVE	PHD	DR	MSIR
0.1	5	100	0.4637 (0.21)	0.6918 (0.23)	0.7916 (0.20)	0.4804 (0.20)	**0.3872** (0.21)
		200	0.3169 (0.13)	0.4727 (0.24)	0.7833 (0.19)	0.2761 (0.11)	**0.1992** (0.09)
		500	0.1842 (0.07)	0.1944 (0.09)	0.7824 (0.21)	0.1800 (0.07)	**0.1070** (0.04)
		1000	0.1290 (0.05)	0.1218 (0.05)	0.7589 (0.21)	0.1071 (0.04)	**0.0739** (0.03)
	10	100	0.7361 (0.18)	0.9856 (0.02)	0.9224 (0.09)	0.7707 (0.17)	**0.7060** (0.17)
		200	0.5172 (0.15)	0.9239 (0.09)	0.9055 (0.10)	0.5639 (0.17)	**0.4185** (0.14)
		500	0.3073 (0.08)	0.5625 (0.20)	0.9033 (0.12)	0.3260 (0.09)	**0.1810** (0.05)
		1000	0.2104 (0.05)	0.2661 (0.08)	0.8852 (0.13)	0.2086 (0.05)	**0.1095** (0.03)
	20	100	0.9040 (0.10)	0.9935 (0.01)	0.9657 (0.05)	0.9327 (0.07)	**0.8414** (0.10)
		200	0.7647 (0.14)	0.9955 (0.01)	0.9608 (0.05)	0.8160 (0.13)	**0.6788** (0.12)
		500	0.4622 (0.09)	0.9940 (0.01)	0.9572 (0.06)	0.5075 (0.11)	**0.3968** (0.08)
		1000	0.3177 (0.06)	0.8119 (0.15)	0.9491 (0.07)	0.3500 (0.06)	**0.2007** (0.04)
0.5	5	100	0.5168 (0.22)	0.7559 (0.22)	0.8035 (0.19)	0.5351 (0.22)	**0.4984** (0.23)
		200	0.3524 (0.15)	0.5484 (0.25)	0.7989 (0.20)	0.3088 (0.14)	**0.2688** (0.13)
		500	0.2031 (0.08)	0.2561 (0.13)	0.7802 (0.21)	0.2080 (0.09)	**0.1264** (0.05)
		1000	0.1418 (0.06)	0.1532 (0.07)	0.7666 (0.22)	0.1219 (0.04)	**0.0831** (0.04)
	10	100	0.7651 (0.17)	0.9873 (0.02)	0.9254 (0.08)	0.8017 (0.17)	**0.7434** (0.17)
		200	0.5709 (0.16)	0.9481 (0.08)	0.9089 (0.10)	0.6298 (0.18)	**0.5249** (0.16)
		500	0.3355 (0.09)	0.7014 (0.20)	0.9002 (0.12)	0.3703 (0.11)	**0.2506** (0.07)
		1000	0.2335 (0.06)	0.3589 (0.12)	0.8867 (0.13)	0.2380 (0.06)	**0.1505** (0.04)
	20	100	0.9285 (0.08)	0.9934 (0.01)	0.9636 (0.05)	0.9395 (0.07)	**0.8674** (0.10)
		200	0.8020 (0.13)	0.9958 (0.01)	0.9604 (0.05)	0.8427 (0.12)	**0.7180** (0.12)
		500	0.5102 (0.10)	0.9948 (0.01)	0.9556 (0.06)	0.5643 (0.11)	**0.4863** (0.09)
		1000	0.3518 (0.06)	0.9032 (0.10)	0.9493 (0.08)	0.3935 (0.08)	**0.2799** (0.05)
1	5	100	**0.6199** (0.23)	0.8253 (0.18)	0.8181 (0.18)	0.6449 (0.23)	0.6587 (0.25)
		200	0.4441 (0.19)	0.7032 (0.23)	0.8096 (0.18)	0.4382 (0.19)	**0.4130** (0.21)
		500	0.2578 (0.11)	0.4097 (0.21)	0.7996 (0.20)	0.2712 (0.14)	**0.1849** (0.08)
		1000	0.1768 (0.07)	0.2352 (0.11)	0.7513 (0.22)	0.1485 (0.06)	**0.1224** (0.05)
	10	100	0.8152 (0.15)	0.9851 (0.02)	0.9158 (0.10)	0.8517 (0.14)	**0.8129** (0.16)
		200	0.6925 (0.17)	0.9714 (0.04)	0.9086 (0.11)	0.7519 (0.16)	**0.6580** (0.17)
		500	0.4197 (0.11)	0.8411 (0.15)	0.9023 (0.12)	0.4848 (0.16)	**0.3690** (0.11)
		1000	0.2852 (0.08)	0.5979 (0.20)	0.8920 (0.13)	0.3008 (0.08)	**0.2184** (0.06)
	20	100	0.9520 (0.05)	0.9926 (0.01)	0.9670 (0.04)	0.9567 (0.05)	**0.9072** (0.08)
		200	0.8870 (0.11)	0.9951 (0.01)	0.9601 (0.05)	0.8993 (0.10)	**0.8144** (0.11)
		500	0.6461 (0.14)	0.9945 (0.01)	0.9594 (0.06)	0.7112 (0.14)	**0.6035** (0.11)
		1000	0.4311 (0.08)	0.9673 (0.04)	0.9537 (0.06)	0.5169 (0.11)	**0.4200** (0.08)

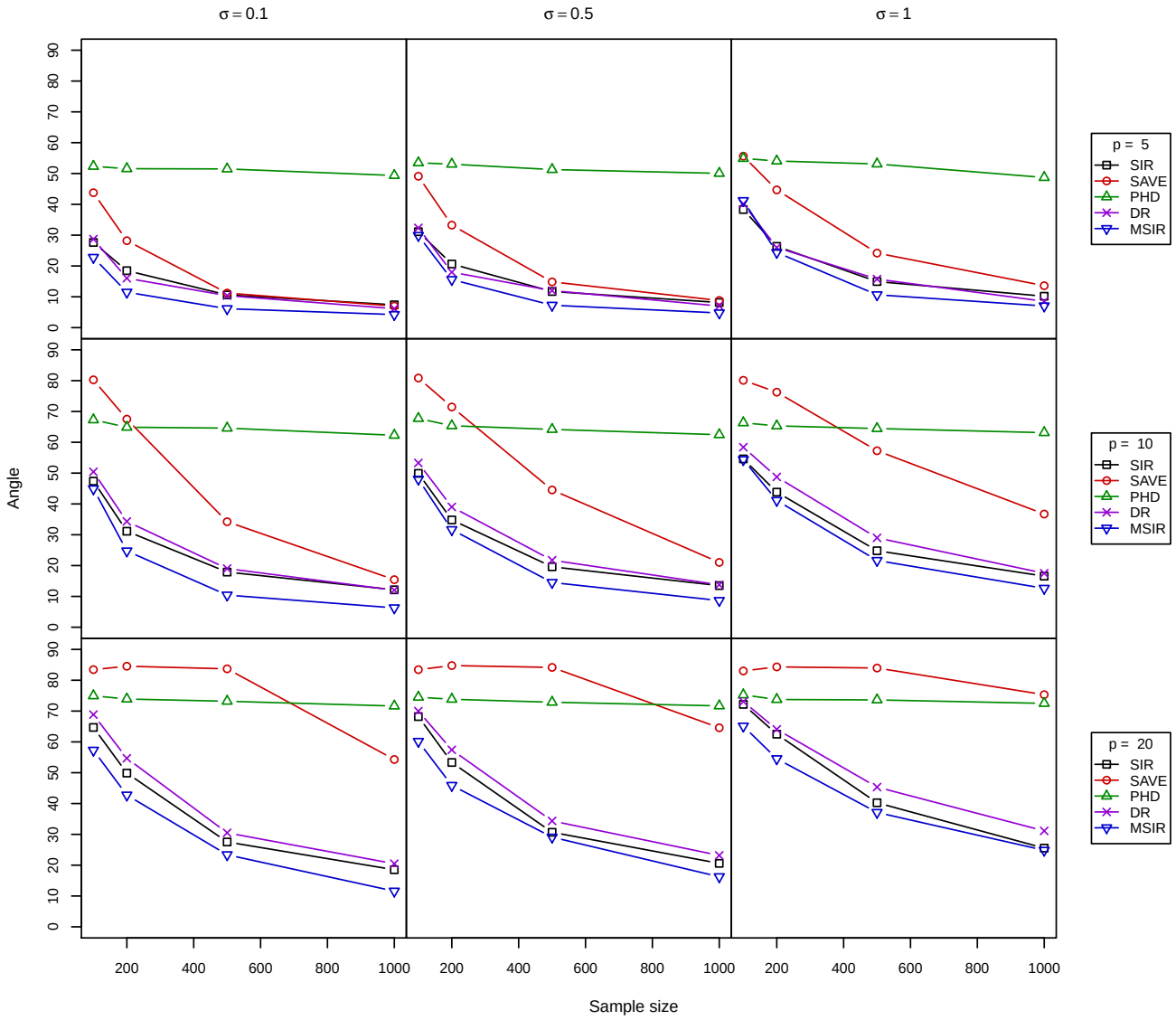

Figure 3: Graphs of simulation results for *model 3*.

Table 4: Simulation results for *model 4*: average distance $\Delta(\hat{\boldsymbol{B}}, \boldsymbol{B})$ with the corresponding standard deviation within parenthesis, based on 500 simulations for different sample sizes (n), number of predictors (p) and correlation coefficients (ρ).

ρ	p	n	SIR	SAVE	PHD	DR	MSIR
0.0	5	100	0.2740 (0.15)	0.4461 (0.24)	0.2714 (0.10)	0.2684 (0.11)	**0.2005** (0.13)
		200	0.1878 (0.08)	0.2104 (0.10)	0.1879 (0.07)	0.1583 (0.06)	**0.0946** (0.04)
		500	0.1139 (0.04)	0.1047 (0.04)	0.1227 (0.05)	0.1245 (0.05)	**0.0417** (0.02)
		1000	0.0806 (0.03)	0.0661 (0.03)	0.0887 (0.03)	0.0661 (0.02)	**0.0246** (0.01)
	10	100	0.4465 (0.18)	0.8808 (0.15)	0.4476 (0.12)	0.4165 (0.15)	**0.3723** (0.15)
		200	0.2870 (0.10)	0.7552 (0.22)	0.3068 (0.08)	0.3049 (0.08)	**0.2199** (0.07)
		500	0.1743 (0.05)	0.2617 (0.08)	0.1924 (0.05)	0.1996 (0.05)	**0.1176** (0.03)
		1000	0.1181 (0.03)	0.1325 (0.03)	0.1375 (0.03)	0.1330 (0.03)	**0.0542** (0.01)
	20	100	0.6913 (0.19)	0.9656 (0.06)	0.6806 (0.12)	0.7425 (0.16)	**0.6111** (0.16)
		200	0.4356 (0.13)	0.9597 (0.07)	0.4838 (0.10)	0.4480 (0.10)	**0.4054** (0.10)
		500	0.2533 (0.05)	0.8952 (0.14)	0.2978 (0.06)	0.2636 (0.05)	**0.2218** (0.05)
		1000	0.1731 (0.03)	0.5114 (0.18)	0.2052 (0.04)	0.2011 (0.04)	**0.1330** (0.03)
0.5	5	100	0.5776 (0.22)	0.5410 (0.21)	0.4093 (0.16)	0.4245 (0.16)	**0.1929** (0.10)
		200	0.4285 (0.17)	0.3143 (0.12)	0.3009 (0.11)	0.2568 (0.09)	**0.0990** (0.04)
		500	0.2665 (0.10)	0.1749 (0.07)	0.2010 (0.08)	0.2139 (0.09)	**0.0508** (0.02)
		1000	0.1871 (0.07)	0.1265 (0.05)	0.1408 (0.06)	0.1067 (0.04)	**0.0310** (0.01)
	10	100	0.7449 (0.18)	0.8849 (0.14)	0.5919 (0.13)	0.6262 (0.14)	**0.4146** (0.17)
		200	0.5915 (0.17)	0.6996 (0.19)	0.4460 (0.11)	0.4873 (0.12)	**0.2110** (0.07)
		500	0.4005 (0.11)	0.3502 (0.09)	0.3073 (0.08)	0.3204 (0.09)	**0.0978** (0.03)
		1000	0.2789 (0.08)	0.2134 (0.06)	0.2219 (0.06)	0.2191 (0.06)	**0.0566** (0.02)
	20	100	0.8997 (0.11)	0.9775 (0.04)	0.8149 (0.09)	0.9025 (0.10)	**0.6443** (0.17)
		200	0.7741 (0.14)	0.9490 (0.08)	0.6560 (0.10)	0.6732 (0.11)	**0.4223** (0.12)
		500	0.5400 (0.11)	0.7931 (0.15)	0.4394 (0.08)	0.4354 (0.08)	**0.2017** (0.04)
		1000	0.3943 (0.07)	0.4667 (0.09)	0.3173 (0.06)	0.3349 (0.07)	**0.1142** (0.03)
0.9	5	100	0.8874 (0.13)	0.7740 (0.17)	0.7204 (0.16)	0.7559 (0.16)	**0.3106** (0.13)
		200	0.7969 (0.17)	0.6269 (0.18)	0.6148 (0.18)	0.5354 (0.16)	**0.1863** (0.08)
		500	0.6608 (0.18)	0.4306 (0.15)	0.4701 (0.16)	0.4746 (0.17)	**0.0988** (0.04)
		1000	0.5238 (0.17)	0.3100 (0.12)	0.3549 (0.13)	0.2775 (0.11)	**0.0672** (0.03)
	10	100	0.9541 (0.06)	0.9583 (0.06)	0.8791 (0.08)	0.9020 (0.08)	**0.6561** (0.17)
		200	0.9144 (0.09)	0.8876 (0.09)	0.7978 (0.11)	0.8212 (0.10)	**0.3597** (0.10)
		500	0.8046 (0.12)	0.6717 (0.13)	0.6406 (0.13)	0.6677 (0.12)	**0.1858** (0.05)
		1000	0.6935 (0.13)	0.4997 (0.12)	0.5096 (0.11)	0.5199 (0.12)	**0.1197** (0.03)
	20	100	0.9862 (0.02)	0.9915 (0.01)	0.9609 (0.03)	0.9856 (0.02)	**0.9031** (0.09)
		200	0.9688 (0.04)	0.9828 (0.02)	0.9182 (0.05)	0.9314 (0.05)	**0.7814** (0.14)
		500	0.9084 (0.06)	0.9179 (0.06)	0.7965 (0.07)	0.7825 (0.08)	**0.3856** (0.09)
		1000	0.8254 (0.07)	0.7593 (0.08)	0.6803 (0.09)	0.6967 (0.08)	**0.2151** (0.05)

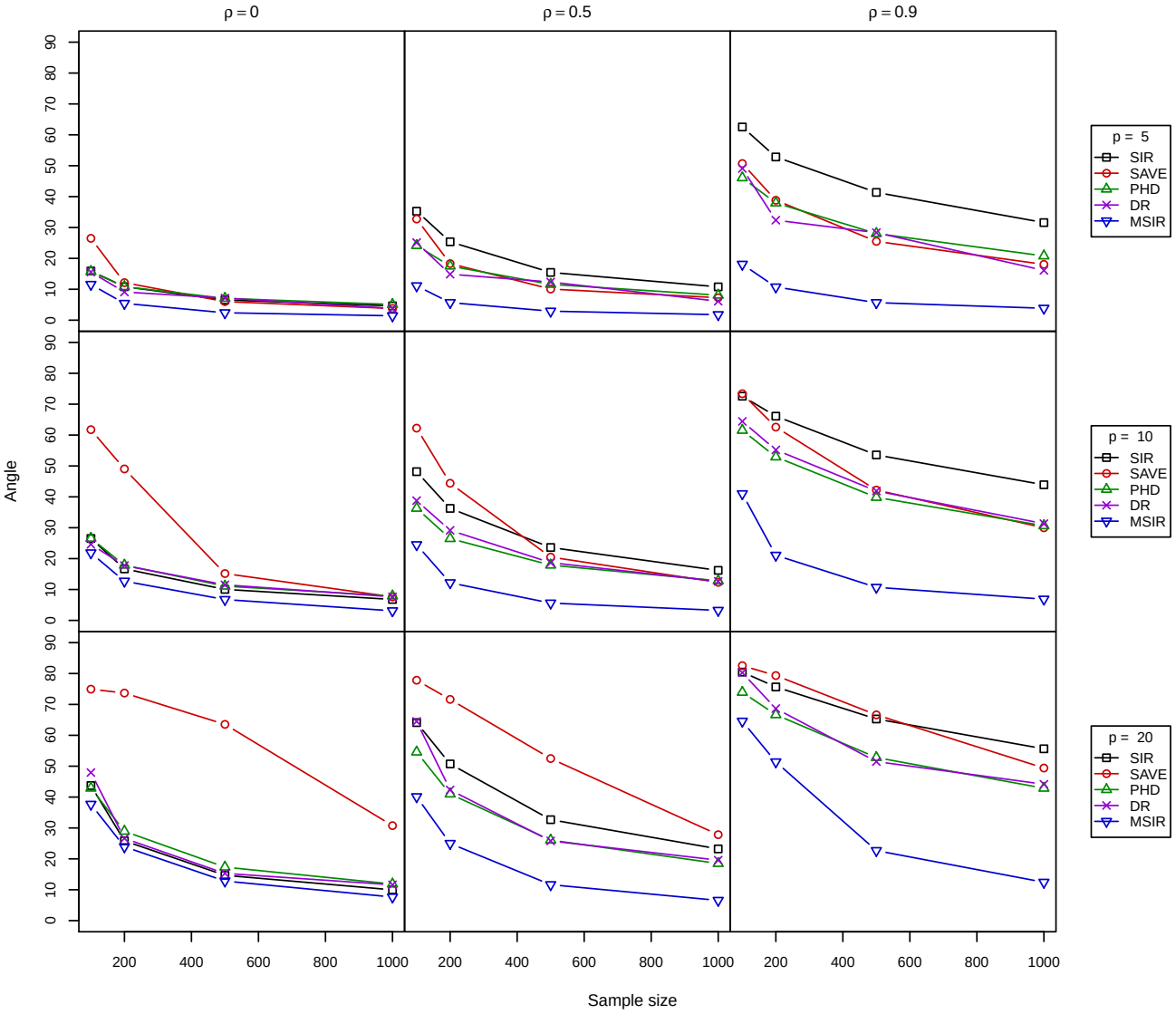

Figure 4: Graphs of simulation results for *model 4*.

Table 5: Simulation results for *model 5*: average distance $\Delta(\hat{\boldsymbol{B}}, \boldsymbol{B})$ with the corresponding standard deviation within parenthesis, based on 500 simulations for different sample sizes (n), number of predictors (p) and parameter value a.

a	p	n	SIR	SAVE	PHD	DR	MSIR
0.0	5	100	0.8835 (0.16)	**0.2969** (0.14)	0.6573 (0.20)	0.3193 (0.14)	0.5285 (0.33)
		200	0.8819 (0.15)	0.1729 (0.07)	0.6492 (0.19)	**0.1671** (0.07)	0.1924 (0.19)
		500	0.8713 (0.16)	0.0990 (0.04)	0.6224 (0.20)	0.1292 (0.06)	**0.0671** (0.02)
		1000	0.8954 (0.14)	0.0651 (0.02)	0.6207 (0.19)	0.0648 (0.02)	**0.0370** (0.01)
	10	100	0.9362 (0.09)	**0.6693** (0.21)	0.7756 (0.13)	0.7038 (0.20)	0.8982 (0.16)
		200	0.9358 (0.09)	**0.3504** (0.12)	0.7492 (0.13)	0.3700 (0.11)	0.7880 (0.28)
		500	0.9361 (0.10)	0.1710 (0.04)	0.7338 (0.13)	0.2080 (0.06)	**0.1236** (0.04)
		1000	0.9409 (0.09)	0.1092 (0.03)	0.7042 (0.13)	0.1259 (0.04)	**0.0800** (0.02)
	20	100	0.9708 (0.04)	0.9452 (0.08)	**0.8677** (0.08)	0.9436 (0.08)	0.9746 (0.04)
		200	0.9686 (0.05)	**0.7435** (0.16)	0.8355 (0.09)	0.7619 (0.17)	0.9683 (0.06)
		500	0.9680 (0.05)	0.3249 (0.06)	0.7970 (0.10)	**0.3148** (0.06)	0.8841 (0.23)
		1000	0.9687 (0.05)	0.1936 (0.03)	0.7698 (0.10)	0.2149 (0.04)	**0.1553** (0.11)
0.5	5	100	0.4764 (0.22)	0.5136 (0.25)	0.6708 (0.20)	**0.3640** (0.16)	0.4570 (0.27)
		200	0.3188 (0.14)	0.2676 (0.13)	0.6630 (0.19)	0.1991 (0.08)	**0.1908** (0.10)
		500	0.1872 (0.07)	0.1325 (0.05)	0.6472 (0.19)	0.1486 (0.06)	**0.0727** (0.03)
		1000	0.1324 (0.05)	0.0853 (0.03)	0.6466 (0.19)	0.0761 (0.03)	**0.0401** (0.01)
	10	100	0.6940 (0.21)	0.8675 (0.15)	0.8003 (0.12)	**0.6888** (0.19)	0.7169 (0.19)
		200	0.4866 (0.17)	0.6876 (0.22)	0.7594 (0.14)	**0.3950** (0.13)	0.4775 (0.16)
		500	0.2839 (0.08)	0.2927 (0.10)	0.7484 (0.13)	0.2363 (0.07)	**0.1781** (0.07)
		1000	0.1940 (0.05)	0.1606 (0.04)	0.7171 (0.14)	0.1520 (0.04)	**0.0871** (0.02)
	20	100	0.8729 (0.14)	0.9730 (0.04)	0.8927 (0.08)	0.8904 (0.12)	**0.8329** (0.13)
		200	0.6974 (0.17)	0.9452 (0.08)	0.8610 (0.09)	0.6883 (0.16)	**0.6680** (0.14)
		500	0.4084 (0.09)	0.8079 (0.17)	0.8185 (0.10)	**0.3520** (0.07)	0.4292 (0.08)
		1000	0.2761 (0.05)	0.4188 (0.12)	0.7893 (0.10)	**0.2444** (0.05)	0.2450 (0.07)
1.0	5	100	**0.2399** (0.10)	0.6654 (0.27)	0.7384 (0.18)	0.3444 (0.16)	0.3042 (0.16)
		200	**0.1604** (0.07)	0.3443 (0.21)	0.6915 (0.19)	0.1976 (0.08)	0.1695 (0.07)
		500	0.1011 (0.04)	0.1283 (0.05)	0.6950 (0.20)	0.1481 (0.07)	**0.0912** (0.03)
		1000	0.0690 (0.03)	0.0742 (0.03)	0.6839 (0.19)	0.0805 (0.03)	**0.0519** (0.02)
	10	100	**0.3845** (0.13)	0.9645 (0.06)	0.8183 (0.13)	0.5292 (0.18)	0.4630 (0.15)
		200	**0.2419** (0.07)	0.9441 (0.09)	0.8056 (0.13)	0.3593 (0.12)	0.2765 (0.08)
		500	**0.1456** (0.04)	0.4336 (0.20)	0.7800 (0.13)	0.2267 (0.07)	0.1542 (0.04)
		1000	**0.1028** (0.03)	0.1549 (0.05)	0.7589 (0.13)	0.1466 (0.04)	0.1054 (0.03)
	20	100	0.6244 (0.17)	0.9880 (0.02)	0.9085 (0.07)	0.7681 (0.16)	**0.5988** (0.13)
		200	**0.3694** (0.08)	0.9903 (0.01)	0.8876 (0.08)	0.4958 (0.12)	0.4006 (0.08)
		500	**0.2170** (0.04)	0.9908 (0.02)	0.8474 (0.09)	0.2962 (0.06)	0.2424 (0.04)
		1000	**0.1494** (0.03)	0.9469 (0.07)	0.8157 (0.10)	0.2262 (0.05)	0.1634 (0.03)

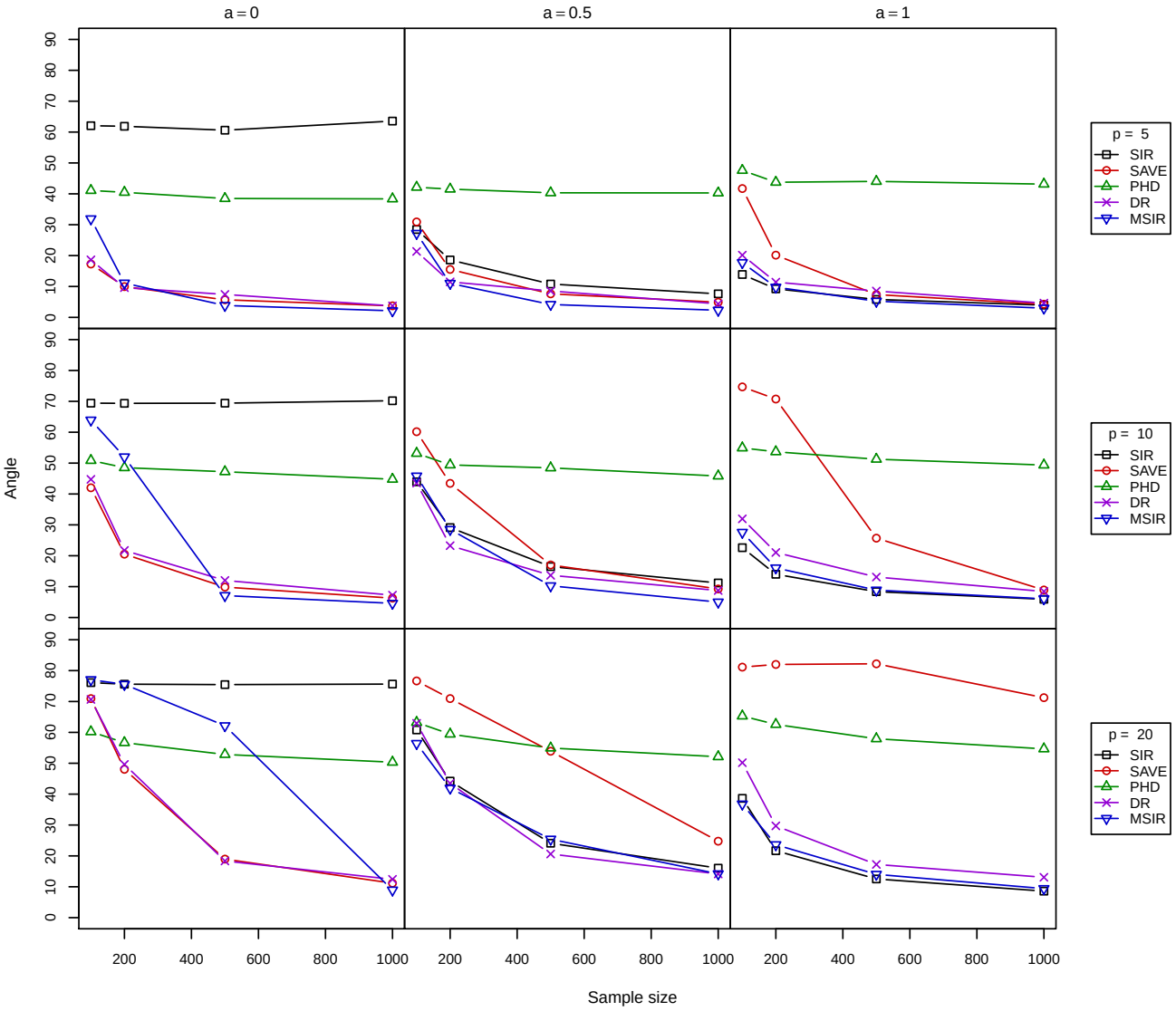

Figure 5: Graphs of simulation results for *model 5*.

5. Data analysis examples

5.1. Chicago air pollution data

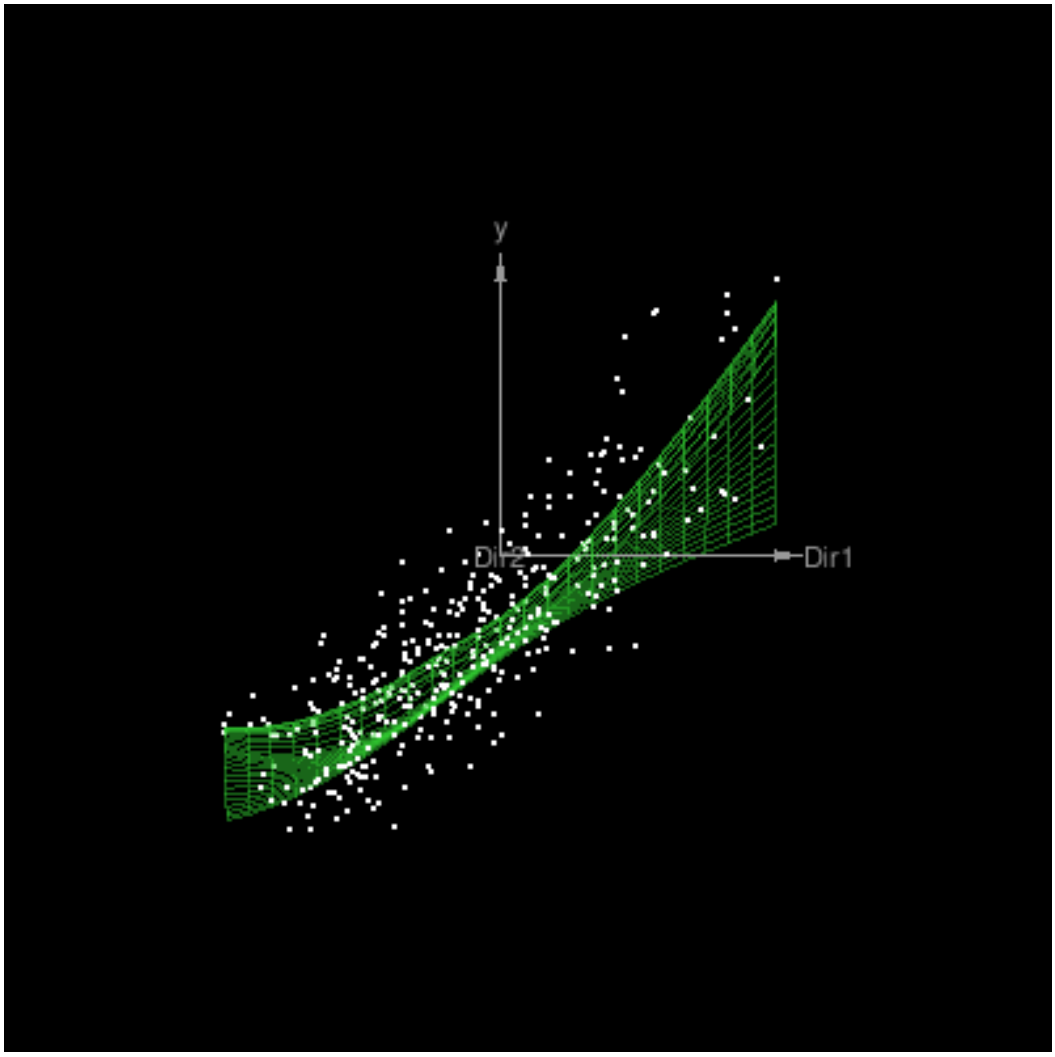

Figure 6: Spin plot of ground level ozone concentration (Y) versus first two estimated MSIR directions, with a smooth function for the mean as reference. If you are viewing this in Acrobat, click on the image to see an animation.

5.2 Pen digit data

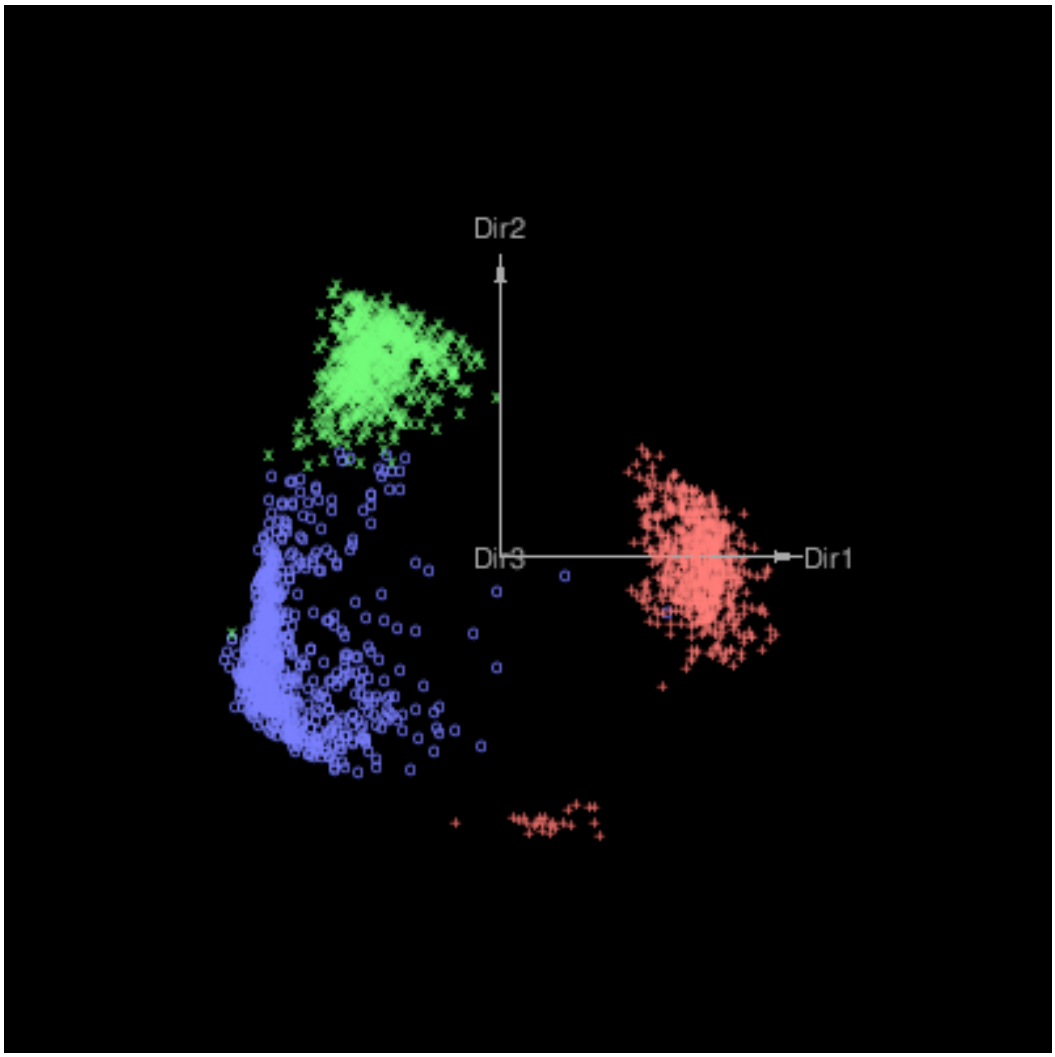

Figure 7: A rotating 3D plot of handwritten digits projectd along first three MSIR directions, with points marked according to digit type: ∘ = 0, × = 6, + = 9. If you are viewing this in Acrobat, click on the image to see an animation.