A REPRESENTATION FOR THE DERIVATIVE WITH RESPECT TO THE INITIAL DATA OF THE SOLUTION OF AN SDE WITH NON-REGULAR DRIFT

OLGA ARYASOVA

We consider a multidimensional SDE with an identity diffusion matrix and a drift vector being a vector function of bounded variation.

According to [8] there exists a unique strong solution to such an equation.

It is well known that if the drift vector is continuously differentiable and its derivative is bounded then the SDE generates a flow of diffeomorphisms. It turns out that this condition can be essentially reduced [5], and a flow of diffeomorphisms exists in the case of possible unbounded Hölder continuous drift vector. Recently the Sobolev differentiability of the solution to an SDE was proved in [3, 4, 6, 7] under rather weak assumptions on the drift. The methods of these works differ from ours. In particular, the Malliavin calculus is used in [6, 7]. The condition we impose on the drift is more restrictive then that of [3, 7], but it allows us to obtain an explicit representation for the derivative in terms of intrinsic parameters of the initial equation. This representation is a natural generalization of the expression for the derivative in the smooth case.

In the one-dimensional case [1, 2] the derivative was represented via the local time of the process. It is well known that the solution does not have a local time at a point in multidimensional situation. We use continuous additive functionals for the representation of the derivative. This method can be considered as a generalization of the local time approach to the multidimensional case.

We show that the derivative is a solution of an integral equation. Our method is likely to can be used in the case of non-constant diffusion.

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Institute of Geophysics, National Academy of Sciences of Ukraine, Palladin pr., 32, Kiev-142, Ukraine
E-mail address: oaryasova@gmail.com