Exploiting Multiple Outcomes in Bayesian Inference for Causal Effects with Intermediate Variables\footnote{Presented at the first meeting of the FIRB (“Futuro in ricerca” 2012) project “Mixture and latent variable models for causal-inference and analysis of socio-economic data”, Perugia (IT), March 15-16, 2013}

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Abstract

In both experimental and observational studies, the causal effects of interest are often defined in terms of confounded intermediate variables, i.e., post-treatment variables potentially affected by the treatment and also affecting the response. Therefore, treatment comparisons often need to be adjusted for such intermediate variables to draw valid causal inferences. Examples of intermediate variables include noncompliance with the protocol, surrogate endpoints, unintended missing outcome data, truncation of outcome by “death”, mediating variables channelling part of the treatment effect, and combinations of these variables.

Discussion of causal inference in this article is carried out under the potential outcome framework, also known as the Rubin Causal Model (RCM) \cite{Rubin1974,Rubin1978}. Under the RCM the key organizing principle for addressing causal inference problems with intermediate variables is based on the concept of principal stratification (PS) \cite{FrangakisRubin2002}. A PS with respect to an intermediate variable is a cross-classification of units into latent classes defined by the joint potential values of that intermediate variable under each of the treatments being compared. A principal stratum consists of units having the same joint intermediate potential outcomes and so is not affected by treatment assignment. Therefore, comparisons of potential outcomes under different treatment levels within a principal stratum - the principal causal effects (PCEs), are well-defined causal effects in the sense of Rubin \cite{Rubin1978}.

Unfortunately, since at most one potential outcome is observed for any unit, principal strata are generally latent, so that inference on PCEs is not straightforward. There are two streams of work in the existing literature regarding this: (1) deriving large-sample nonparametric bounds for
the causal effects under minimal structural assumptions (e.g., [Manski, 1990] [Zhang and Rubin, 2003]); (2) specifying additional structural or modeling assumptions, such as exclusion restrictions or monotonicity-type assumptions, to identify PCEs and conducting sensitivity analysis to check the consequences of violations to such assumptions (e.g., [Sjölander et al., 2009] [Schwartz et al., 2012]). In this article, we introduce an alternative approach of utilizing multiple outcomes in model-based analysis to improve estimation of PCEs.

Multivariate analysis is beneficial for two reasons. First, models used in PS are inherently mixture models; recent work on mixture models shows that with correct model specification, multivariate analyses result in more precise inferences ([Mercatanti et al., 2012]). Second, some key substantive structural assumptions, such as exclusion restriction, may be more plausible for secondary outcomes than the primary one. Restrictions on secondary outcomes reduce the parameter space of the joint distribution of all outcomes and in turn the marginal distribution of the primary one ([Mealli and Pacini, 2012]).

However, the additional information provided by secondary outcomes is obtained at the cost of having to specify more complex multivariate models, which may increase the possibility of misspecification. Therefore, model diagnostics are crucial in the multivariate analysis and we develop model checking procedures via posterior predictive checks in this article.

We use a Bayesian approach to inference. In the Bayesian paradigm, inferences are based on the posterior distribution of the causal estimands defined as functions of observed and unobserved potential outcomes, or sometimes as functions of model parameters. This leads to at least two inferential advantages. First, the Bayesian approach provides a refined map of identifiability, clarifying what can be learned when causal estimands are intrinsically not fully identified, but only weakly identified in the sense that their posterior distributions have substantial regions of flatness ([Imbens and Rubin, 1997]). In particular, issues of identification are different from those in the frequentist paradigm because with proper prior distributions, posterior distributions are always proper. However, unless the prior distributions are extremely informative, weak identifiability is still reflected in the posterior distribution in terms of wide regions of flatness and/or multiple modes.

Second, in a Bayesian setting, the effect of relaxing or maintaining assumptions can be directly checked by examining how the posterior distributions for causal estimands change, therefore serving as a natural framework for sensitivity analysis. Moreover, the Bayesian framework allows one to quantify the impact on the causal estimates when there is a diversion from these assumptions.

The primary aim of the paper is to combine the benefits coming from using a multivariate analysis with the inferential advantages of the Bayesian approach in order to improve inference (i.e., reduce posterior uncertainty) about the treatment effects on the outcome of primary interest. Simulation studies are performed to illustrate the potential gains in identifiability of jointly modelling more than one outcome. The method is applied to evaluate the causal effect of a job training program on trainees’ depression.

As a general message, this article stresses the importance of exploiting the relationships of the outcome of primary interest with other (secondary) outcomes, instead of conducting the analysis on each outcome separately.
References


