Optimal Monetary Policy with Wealth Effect and Cost Channel

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February 4, 2009

Abstract

The principal aim of this paper is to formally explore optimal monetary policy in an environment that differs from the standard New Keynesian one in two aspects. On the one hand, we suppose that agents do not have an infinite time horizon. In a context of *finite life*, risky assets are net wealth and affect the dynamics of aggregate consumption and inflation. On the other hand, we introduce a *cost channel* of monetary policy relating marginal cost and the interest rate.

From the equilibrium determinacy and stability under adaptive learning viewpoint we show that financial stability is an important target to achieve the economy's stability. In particular, we demonstrate that if the central bank adds financial stability to its traditional objectives of inflation and output stabilitzation, optimal monetary rule under discretion yields determinacy and learnability.

Keywords: learning, cost channel, monetary policy

JEL classiffication: E4, E5

1 Introduction

Whether monetary policy should respond to asset prices or not is a research question still lacking a clear answer.

Japan equity bubble in the late 1980's, the *dot.com* bubble, South-East Asia crisis in 1997-1998 and the more recent *subprime* crisis demonstrate that financial instability can affect dangerously not only the system where it originates, but also it can have repercussions worldwide. Furthermore some empirical analysis, among others Rigobon and Sack (2003) have provided evidence, suggesting that central banks sometimes respond to financial markets' fluctuations.

*Email: roberta.cardani@unicatt.it. We are grateful to Domenico Delli Gatti, Seppo Honkapohja and Michele Berardi for their useful suggestions during the development of the idea that this paper concerns.

In particular they show that the Federal Reserve does react with vigor to stock market oscillations.

Nevertheless, there is no clear consensus in the literature regarding its role in the inflation process.

Even if it is widely recognized that booms and busts in financial markets are among the main determinants of macroeconomic fluctuations, there is no agreement in the profession on the appropriate response of the central bank and its role in the inflation process.

On one hand, Bernanke and Gertler (2001b) suggest that stock market investors are not endowed with any private information that is not available to the central bank. Following this assumption, they demonstrate that inflation stabilization actually already implies asset stabilization. Bullard and Schaling (2002) and Carlstrom and Fuerst (2001, 2007) show that incorporating asset prices in a Taylor-type interest rate rule does not improve economic performance but it enlarges the indeterminacy area as the weight on asset prices increases. In this sense, reacting systematically to irregular and volatile asset price changes could be destabilizing *de facto* for the macroeconomy.

On the other hand, Cecchetti et al. (2000), Dupor (2003, 2005) and Bordo and Jeanne (2002) Girlchrist and Saito (2006) claim that central banks obtain some benefits from including asset prices in the reaction function because in this way central banks can limit the potential costs (in terms of output) when the bubble bursts.

The principal aim of this paper is to investigate the implication of the optimal monetary policy when asset prices affect the real economy through aggregate demand, in accordance with the findings of Goodhart and Hofmann (2000, 2003).[1] In our framework stock prices pertub the aggregate demand through a wealth effect which impacts on consumption due to uncertain lifetimes of the households. This provides the reason why the central bank should pay attention to asset prices.

As in Nisticò (2005) and Airaudo et al. (2007), we consider an overlapping generations model (Yaari, 1965, Blanchard, 1985), in which agents face up a constant probability to die and therefore are not able to smooth consumption, as in the case of infinitely lived agents. Compared with the papers mentioned above, the novel feature of our model is the modified setting characterized by the *cost channel* of the monetary transmission mechanism.

Recent contributions to the cost channel literature are due to Barth and Ramey (2001), Christiano et al. (2005), Ravenna and Walsh (2006) and Chowdhury, Hoffmann and Schabert (2006), that integrate a cost channel in an otherwise standard New Keynesian model. This literature is based on the assumption that firms must pay the factors of production before they get revenues from selling their products. Consequently they borrow from financial intermediaries the funds needed to finance production.

In this framework the nominal interest rate is a determinant of the real

[1] Goodhart and Hofmann (2000) detect empirically the link between output growth, credit aggregates and asset price movements in a number of industrialized economies.

marginal cost and hence of inflation: a higher interest rate translates into a higher production cost and inflation that offsets at least in part the reduction of inflation due to the decline in aggregate demand. A link is therefore established between the financial sector and the supply side of the economy.

In this paper we explore the optimal monetary policy under discretion from learning viewpoint. Following Sargent (1999) and Evans and Honkapohja (2001) we assume that agents form their expectations by means of an *adaptive learning algorithm*, such as recursive least squares, based on the data produced by the economy. We show that if the policy maker follows an optimal fundamental-based rule, small expectational errors by private agents drive the economy away from the rational expectations equilibrium (REE).

Then, we consider the case of an expectations-based rule under discretion. In this case the central bank should respond to asset price movements when agents have a finite planning horizon. Our result is that even if the central bank aims at financial stability and commits to preventing any financial market collapse, the REE is determinate and thus learnable just in the case of finitely lived agents.

The paper is organized as follows: section 2 derives a DSGE model with wealth effect and cost channel. In section 3 we study optimal monetary policy under discretion and consider the implication for learning, i.e. we derive the determinacy and E-stability conditions. Moreover, in section 4 we analyze the case in which the policymaker targets explicitly asset price stability. Finally, section 5 concludes.

1.1 The model

We analyze a New Keynesian economy augmented by a cost channel mechanism of monetary policy transmission within the context of a discrete time Blanchard-Yaari finite horizon model.

The economy consists of a large number of identical finitely lived households that consume differentiated goods, accumulate assets and supply labour. Agents face a constant probability of death in each period. Since these agents have no bequest motive, a portion of the outstanding stock of risky assets will be net wealth, as shown in Airaudo et al. (2007). It follows that fluctuations in asset prices affect the demand side – hence inflation – through a wealth effect.

On the supply side, we consider two sectors: under monopolistic competition, the wholesale sector produces a continuum of differentiated intermediate goods which are sold to the retail sector that produces the finished goods.

Finally, the central bank sets the nominal interest rate optimally, i.e. with the goal to minimize its loss function.

1.1.1 Households

Households have identical preferences and face the same constant probability to die γ. Therefore $1/\gamma$ is the actual planning horizon. By assumption population is constant and is normalized to 1. Therefore, in each period a fraction γ of agents (or members of the household) dies while a fraction $1-\gamma$ survives. γ can be conceived of as the objective probability of existing market [2].

As in Blanchard, households do not show inter-generational altruism so that there is no bequest motive to save. Therefore they sell contingent claims to their wealth to insurance companies and sign a contract to make (or receive) a payment contingent on their death. At the beginning of each period, insurance companies gather financial assets from the deceased members of an household and pay a fair premium to survivors. The zero profit condition in the insurance industry requires a premium payment of $\frac{\gamma}{1-\gamma}$ per unit of asset held by survivors. As a consequence, the gross return on the insurance contract, incorporated in the flow budget constraint, is given by $1+\frac{\gamma}{1-\gamma}=\frac{1}{1-\gamma}$.

For simplicity, labor income, profits and lump-sum taxes are age-independent, as in Cushing (1999). The preferences of household j born in period s – i.e. of generation s – are defined over leisure $1-N_{s,t}(j)$, where $N_{s,t}(j)$ is the time spent at work and a consumption good $C_{s,t}(j)$. We also assume that the preferences are perturbed by exogenous stochastic shocks shifting the marginal utility of consumption τ_t and the marginal disutility of labor χ_t.[3]

Each household j maximizes the expected present value of utility at time t:

$$E_t \sum_{t=i}^{\infty} \beta^{t-i} (1-\gamma)^{t-i} \left\{ \Gamma_t \ln C_{s,t}(j) + \Upsilon_t \ln \left[1 - N_{s,t}(j)\right] \right\} \tag{1}$$

where the actual discount factor is the product of the intertemporal discount factor β and the probability of survival $1-\gamma$.

Financial wealth can take two forms: a state-contingent bond issued by the government and a risky asset issued by the wholesale firms. Therefore at the beginning of time t, an agent of generation s has financial wealth $A_{s,t}(j)$,consisting of $B_{s,t}(j)$, the nominal government bond, and a portfolio of equities, each one denoted by $V_{i,t}(j)$, issued by the $i-th$ wholesale firm, whose real price in period t is $Q_t(i)$, which pays a stochastic dividend $D_t(i)$:

$$A_{s,t}(j) \equiv B_{s,t}(j) + P_t \int_0^1 \left[Q_t(i) + D_t(i)\right] V_{i,t}(j) di \tag{2}$$

In each period t households' resources are given by labor income W_t deducted the nominal lump sum taxes $T_{s,t}(j)$ and by the return on nominal total financial wealth $\dfrac{A_{s,t}(j)}{1-\gamma}$.[4] Therefore, the flow budget constraint of agent j of generation s is :

$$P_t C_{s,t}(j) + E_t \left[F^s_{t,t+1} B_{s,t+1}(j)\right] + P_t \int_0^1 Q_t(i) V_{i,t+1}(j) di \leq W_t N_{s,t}(j) - P_t T_{s,t}(j) + \frac{A_{s,t}(j)}{1-\gamma} \tag{3}$$

[2] Blanchard (1985,p.225) put forward an alternative interpretation according to which γ is the probability that "family ends – i.e. that members of the family die without children".

[3] As in Nisticò (2005) we assume that those shocks are defined as $\Gamma_t = e^{\tau_t}$ and $\Upsilon_t = e^{\chi_t}$.

[4] We recall that $\frac{1}{1-\gamma}$ is the return on the insurance contract mentioned above.

where $E_t\left[\digamma^s_{t,t+1}B_{s,t+1}(j)\right]$ is the expected value of bondholding at the beginning of period $t+1$ – $\digamma^s_{t,t+1}$ is the stochastic discount factor or asset-pricing kernel common across cohorts– and $\int_0^1 Q_t(i)V_{i,t+1}(j)di$ is the value of shareholding at the beginning of period $t+1$.

Maximization of expected utility (1) in $t=0$ subject to the (3), yields the following FOCs:

$$\begin{aligned} \frac{P_t C_{s,t}(j)\tau_{t+1}}{P_{t+1}C_{s,t+1}(j)\tau_t} &= \frac{1}{\beta}E_t\left\{\digamma^s_{t,t+1}\right\} && (4) \\ \chi_t C_{s,t}(j) &= \frac{W_t\tau_t}{P_t}\left[1-N_{s,t}(j)\right] && (5) \\ P_t Q_t(i) &= E_t\left\{\digamma^s_{t,t+1}P_{t+1}\left[Q_{t+1}(i)+D_{t+1}(i)\right]\right\} && (6) \end{aligned}$$

where (4) is the Euler consumption equation expressed in terms of asset price, (5) represents the consumption/leisure efficiency condition and (6) is the no-arbitrage condition concerning asset prices.

The expected price $E_t\left\{\digamma^s_{t,t+1}\right\}$ of a one-period *riskless* asset is the reciprocal of the (gross) short term interest rate r_t, as in Woodford (2003):

$$E_t\left\{\digamma^s_{t,t+1}\right\} = \frac{1}{(1+r_t)} \tag{7}$$

Combining (4) with (7) we obtain:

$$1 = (1+r_t)\beta E_t\left\{\frac{\tau_{t+1}C_{s,t}(j)}{\tau_t C_{s,t+1}(j)}\frac{P_t}{P_{t+1}}\right\} \tag{8}$$

that is the familiar Euler consumption equation.

At the optimum the flow budget constraint (3) holds with equality in each period. We impose also the transversality condition:

$$\lim_{k\to\infty} E_t\left\{\digamma^s_{t,t+k}\left(1-\gamma\right)^k A_{s,t+k}(j)\right\} = 0 \tag{9}$$

that avoids a strategy of unlimited borrowing to support unlimited consumption in the last period conditionally to the probability of death.

Solving the optimal consumption plan forward, using the portfolio equilibrium condition (6) and the budget constraint (3) and imposing the no Ponzi game condition (9), we obtain that individual consumption is a linear function of financial wealth $A_{s,t}$ and human wealth $h_{j,t}$:

$$P_t C_{s,t}(j) = \frac{1}{\Omega_t}\left[A_{s,t}(j)+h_t(j)\right] \tag{10}$$

where

$$h_t(j) \equiv E_t\left\{\sum_{k=0}^{\infty}\digamma^s_{t,t+k}\left(1-\gamma\right)^k\left[W_{t+k}N_{t+k}\left(j\right)-P_{t+k}T_{t+k}\left(j\right)\right]\right\} \tag{11}$$

is the discounted sum of expected future labor income net of taxes and

$$\Omega_t \equiv E_t \left\{ \sum_{k=0}^{\infty} F^s_{t,t+k} (1-\gamma)^k \left(\frac{\tau_{t+1}}{\tau_t} \right) \right\}$$

is the reciprocal of the time-varying marginal propensity to consume out of financial and human wealth[5].

Finally the market clearing condition implies that $Y_t = C_t + F_t$ where F_t is Government expenditure. As in Galì (2003), we consider the case in which Government expenditure is a stochastic fraction of current output[6] $F_t = \varrho_t Y_t$ and is totally financed by lump sum taxation:$F_t = T_t$. In other words, we consider a balanced budget.

1.1.2 Aggregation across Cohorts

We define the aggregate state of variable X at date t as a weighted average of the cohorts' states from the beginning of times up until t, where the weights n are given by the cohorts' sizes, which in turn depend on the probability of survival:

$$X_t = \sum_{j=-\infty}^{t} n_{j,t} X_{j,t} = \sum_{j=-\infty}^{t} \gamma (1-\gamma)^{t-j} X_{j,t} \tag{12}$$

Note that t refers to the agents born in period $t-1$.

Since the equilibrium conditions specific to each cohort j are linear, aggregation through (12) preserve the functional form of the cohorts' counterparts[7]:

$$C_t = \frac{\tau_t W_t}{\chi_t P_t} [1 - N_t] \tag{13}$$

$$(\Omega_t - 1) P_t C_t = \gamma E_t \{F_{t,t+1} A_{t+1}\} + (1-\gamma) E_t \{F_{t,t+1} \Omega_{t+1} P_{t+1} C_{t+1}\} \tag{14}$$

$$A_t \equiv \left[B_t + P_t \int_0^1 Q_t(j) V_{j,t+1}(j) dj \right] \tag{15}$$

Equation (14) captures the impact of financial wealth on consumption. The probability to survive affects the degree of smoothing in the inter-temporal path of aggregate consumption, because agents cannot fully smooth over time the effects of the shocks. As a consequence, stock market booms or busts lead to fluctuations in current consumption. If γ goes to zero, i.e. agents are infinitely lived, then the wealth effect disappears and we obtain the usual Euler condition.

[5] In absence of taste shocks the propensity to consume out of financial and human wealth is simply $\Omega = 1 - \beta(1-\gamma)$. For details see Piergallini (2006) and Airaudo, Nisticò and Zanna (2007).

[6] We assume that ϱ_t is such that $f_t = \rho_f f_{t-1} + u_{f,t}$ with $\rho_f \in [0,1)$ and $u_{f,t} \sim \left(0, \sigma_f^2\right)$.

[7] See the details in the appendix.

1.1.3 Firms

The supply side of our economy consists of two sectors: a retail sector that operates under perfect competition to sell the final goods to households and a wholesale sector which operates under monopolistic competition to produce a continuum of differentiated intermediate goods.

Retail sector In each period t the retail firms employ $Y_t(i)$ units of goods $i \in [0,1]$ bought at the nominal price $P_t(i)$ to produce the composite good Y_t through the CRS technology:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\theta_t - 1}{\theta_t}} di \right]^{\frac{\theta_t}{\theta_t - 1}} \tag{16}$$

where $\theta_t > 1$ represents the elasticity of demand for each good.

As in Steinsson (2003) and Ireland (2004), the elasticity of substitution between intermediate goods is time-varying, meaning that the substitutability is constantly changing. Therefore also the market power of each firm and its desired markup over the marginal cost is changing.

Following Airaudo et al. (2007) we assume also that θ_t follows a log-stationary autoregressive process:

$$\ln \theta_t = (1 - \rho_\theta) \ln \theta + \rho_\theta \theta_{t-1} + \varepsilon_{\theta,t}$$

where $\theta_t > 1$, $0 < \rho_\theta < 1$ and $\varepsilon_{\theta,t} \sim WN(0, \sigma_\varepsilon^2)$. As we will see, this assumption generates a cost push shock that augments the traditional New Keynesian Phillips Curve.

Wholesale sector The wholesale $i - th$ firm maximizes profits by choosing an optimal point on the demand curve of intermediate good i

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\theta_t} Y_t \tag{17}$$

where $\frac{P_t(i)}{P_t}$ is the relative price and

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta_t} di \right]^{\frac{1}{1-\theta_t}} \tag{18}$$

is the aggregate price index for consumption.

The wholesale $i - th$ firm hires $N_t(i)$ units of labor in a competitive labor market and produces according to the following constant returns to scale technology:

$$Y_t(i) = Z_t N_t(i) \tag{19}$$

where Z_t is a stochastic productivity shock with mean one.

As in Ravenna and Walsh (2006), we assume that before selling output, firms must borrow an amount $W_t N_t(i)$ from banks at the gross nominal interest rate $R_t = 1 + r_t$, so the nominal labour cost is $R_t W_t$. By construction the interest rate on loans is equal to the interest rate on Government bonds.

The real marginal cost is $\varphi \equiv \dfrac{R_t W_t}{P_t Z_t}$ and is uniform across firms. The relative price of the $i-th$ good is $\frac{P_t(i)}{P_t} = \Phi_t \frac{R_t W_t}{P_t Z_t}$ where $\Phi_t = \frac{\theta_t}{\theta_t - 1} > 1$ is the time-varying markup. In the long run equilibrium $\frac{P_t(i)}{P_t} = 1$ so that the marginal cost turns out to be the reciprocal of the markup $\Phi > 1$:

$$\varphi_t \equiv \frac{R_t W_t}{Z_t P_t} = R_t S_t = \underbrace{\frac{\theta - 1}{\theta}}_{1/\Phi} \tag{20}$$

where $S_t = W_t / P_t Z_t$ is the share of labor in aggregate income. A decrease of the marginal cost is associated with an increase in monopoly power.

Following Calvo (1983), we assume that in each period some firms are unable to adjust their price. $1 - \omega$ is the probability that a firm optimally adjusts its price – and the fraction of firms which are adjusting their prices. The fraction ω of firms that do not adjust their prices, simply update their previous price by the steady-state inflation rate. The parameter ω captures the degree of nominal price stickiness: as it becomes smaller, the model becomes closer to perfect price flexibility.

Following the usual procedure we get the inflation adjustment equation

$$\pi_t = \widetilde{\beta} E_t \pi_{t+1} + \kappa \widehat{\varphi}_t + u_t \tag{21}$$

where $\widetilde{\beta} = \frac{\beta}{1+\psi}$, $\psi = \gamma \frac{1-\beta(1-\gamma)\Omega}{(1-\gamma)PC}$ and $\kappa = \frac{(1-\omega)(1-\omega\beta)}{\omega}$.

$\widehat{\varphi}_t$ represents the log-deviation of the marginal cost around its steady state while the cost-push shock u_t is defined as:

$$u_t = (1 - \Phi) \frac{(1-\omega)(1-\omega\beta)}{\omega} \ln \frac{\theta_t}{\theta}$$

where $\Phi \equiv \frac{\theta}{\theta - 1}$ is the steady state markup. u_t can be than approximated as an AR (1) process:

$$u_t = \rho_{\theta_t} u_{t-1} + \theta_{u,t} \sim iid(0, \sigma_u^2)$$

We can establish a relation between the degree of competition θ_t and the supply shock u_t: if the market power decreases, i.e. θ_t goes up, the gross markup decreases with a negative effect on inflation as all firms lower their prices.

1.2 Equilibrium

Aggregating across firms and considering the demand for the intermediate good $Y_t(i)$ (17), we have:

$$Z_t N_t = Y_t \int_0^1 \left[\frac{P_t(i)}{P_t} \right]^{-\theta_t} di$$

where $N_t \equiv \int_0^1 N(i)di$ is total hours worked and $\int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\theta_t} di$ is the measure of price dispersion over a continuum of wholesale firms.[8]

The stock of equities outstanding is normalized to 1, i.e. $\int_0^1 V_t(i)di = 1$ for $i \in (0,1)$ and the total real dividend payments and the aggregate real stock price index are defined as an integral over the continuum of firms, i.e. $D_t \equiv \int_0^1 D_t(i)di$ and $Q_t \equiv \int_0^1 Q_t(i)di$.

The demand side of the economy is given by the following equations:

$$\begin{aligned} Y_t &= C_t + F_t = C_t + \varrho Y_t & (22)\\ P_t Y_t &= N_t W_t + P_t D_t & (23)\\ C_t &= \frac{\tau_t W_t}{\varkappa_t P_t}(1 - N_t) & (24)\\ (\Omega_t - 1) C_t &= \gamma Q_t + (1-\gamma) E_t \left\{ \Gamma_{t,t+1} \frac{P_{t+1}}{P_t} \Omega_{t+1} C_{t+1} \right\} & (25) \end{aligned}$$

and the two asset pricing equations:

$$\begin{aligned} Q_t &= E_t \left\{ \Gamma_{t,t+1} \frac{P_{t+1}}{P_t} (Q_{t+1} + D_{t+1}) \right\} & (26)\\ 1 &= (1 + r_t) E_t \{\Gamma_{t,t+1}\} & (27) \end{aligned}$$

Note that according to (25) consumption is affected by the dynamics of asset prices as defined in (26). If the probability of death $\gamma = 0$, we go back to the infinitely-lived consumers and we obtain the traditional Euler condition.

From equations (26) and (27) we determine the one period riskless nominal interest rate r_t[9]:

$$1 + r_t = \frac{E_t \left\{ \Gamma_{t,t+1} \frac{P_{t+1}}{P_t} \left(\frac{Q_{t+1} + D_{t+1}}{Q_t} \right) \right\}}{E_t \{\Gamma_{t,t+1}\}} \tag{28}$$

[8] Note that the price dispersion's measure is not considered in the log-linearized equilibrium, since its log value is of second order.

[9] For the sake of simplicity we choose not to consider the "*irrational exuberance*" in the equity premium. See Nisticò (2005) and Smets and Wouters (2002) for this modelling choice.

1.3 Steady state

The log-linearization around the non stochastic zero inflation steady state of the equations (22), (23), (24) and (25) leads to:

$$\begin{aligned}
y_t &= c_t + f_t && (29)\\
w_t - p_t &= c_t + \eta n_t - (\tau_t + \varkappa_t) && (30)\\
y_t &= z_t + n_t && (31)\\
c_t &= \frac{1}{1+\psi} E_t c_{t+1} + \frac{\psi}{1+\psi} q_t - \frac{1}{1+\psi}\left(r_t - E_t \pi_{t+1} - \widetilde{\rho}\right) + \\
&\quad - \left(1+\psi_\tau\right) E_t \Delta\tau_{t+1} + \frac{1+\psi-\rho_f}{1+\psi} f_t && (32)\\
q_t &= \widetilde{\beta} E_t q_{t+1} + \left(1-\widetilde{\beta}\right) E_t d_{t+1} - \left(r_t - E_t \pi_{t+1} - \widetilde{\rho}\right) && (33)\\
d_t &= \frac{Y}{D} y_t - \frac{WNR}{PD}\left(n_t + w_t - p_t + r_t\right) && (34)
\end{aligned}$$

where $\widetilde{\rho} = \log(1+r) = -\log \widetilde{\beta}$ is the net interest rate of its steady state, $\widetilde{\beta} = \frac{1}{1+r_t} = \frac{\beta}{1+\psi}$ is the stochastic discount factor, $\eta = \frac{N}{1-N} = \frac{1}{R_t \Phi(1-\varrho_t)}$ is the inverse of the steady state Frisch elasticity of labor supply, $\psi_\tau = \frac{\psi[\beta(1-\gamma)\rho_\tau]}{(1+\psi)[1-\beta(1-\gamma)\rho_\tau]}$ (where ρ_τ is the the degree of taste shock's persistence), ρ_f is the degree of the fiscal policy persistence and $f_t \equiv -\log\left(\frac{1-\varrho_t}{1-\varrho}\right)$ (where ϱ_t is the fraction of public spending).

Denote $\ln R_t \equiv r_t$, $\ln W_t = w_t$, $\ln P_t = p_t$ and $\ln Z_t = z_t$. As regards the marginal cost φ_t, from (20) follows:

$$\ln \varphi_t = r_t + w_t - p_t - z_t$$

Considering also the equilibrium condition on the labor market (30) and the aggregate resource constraint (29), we obtain:

$$\ln \varphi_t = (1+\eta)\left(y_t - z_t\right) - \left(f_t + \tau_t + \varkappa_t\right) + r_t \qquad (35)$$

When prices are flexible, $\omega = 0$ and in the absence of shocks, the price is set as a constant markup over the nominal marginal cost. As shown in the appendix, $\ln \varphi_t = 0$.

Imposing this condition in equation (35), we get the natural level of output:

$$\widetilde{y}_t^n = z_t + \frac{1}{(1+\eta)}\left(f_t + \tau_t + \varkappa_t\right) - \frac{1}{(1+\eta)} r_t^n \qquad (36)$$

On the other hand, when prices are sticky, $\omega > 0$, output can be different from the natural level. Inflation adjustment is given by (21). Firms do not adjust their prices in each period, but consider the present values of the demand for goods by the agents and the marginal costs' flows. Using the definition (36), the expression for the New Keynesian Phillips curve becomes:

$$\pi_t = \widetilde{\beta} E_t \pi_{t+1} + \kappa (1+\eta) x_t + \kappa \left(r_t - \widetilde{r}_t^n\right) + u_t$$

1.4 Dynamics

Our economy is described by the following equations:[10]

$$\pi_t = \widetilde{\beta} E_t \pi_{t+1} + \kappa (1+\eta) x_t + \kappa (r_t - \widetilde{r}_t^n) + u_t \tag{37}$$

$$y_t = \frac{1}{1+\psi} E_t y_{t+1} + \frac{\psi}{1+\psi} q_t - \frac{1}{1+\psi} (r_t - \widetilde{r}_t^n - E_t \pi_{t+1}) - (1+\psi_\tau) E_t \Delta\tau_{t+1} - \Delta f_{t+1} \tag{38}$$

$$q_t = \widetilde{\beta} E_t q_{t+1} + \left(1 - \widetilde{\beta}\right) E_t y_{t+1}^n - \delta E_t x_{t+1} - \varsigma E_t r_{t+1} - (r_t - \widetilde{r}_t^n - E_t \pi_{t+1}) \tag{39}$$

where $\delta = \left(\frac{2+\eta-\Phi}{\Phi-1}\right)\left(\frac{r}{1+r}\right)$, $\varsigma = \left(\frac{r}{1+r}\right)\left(\frac{1}{\Phi-1}\right)$. Note that $\Phi \in (1, 2+\eta)$ so that the coefficient $\delta > 0$.

The reason why the future output gap enters negatively into the asset price equation (39) is that, as shown in the appendix, expectations on dividends depend negatively on future real marginal costs and thus on future output gaps. It follows that the smaller is the markup Φ, the wider is the magnitude of a reduction in future dividends for a given increase in the future output gap.

r_t^n is defined as the real interest rate consistent with the flexible price output:[11]

$$\widetilde{r}_t^n = \overline{\rho} + \widetilde{r} \widetilde{r}_t^n + \psi_f f_t + \psi_\tau \tau_t - \frac{\psi\varsigma}{(1+\psi)} E_t r_{t+1}^n \tag{40}$$

As shown in the appendix, the impact of demand shocks ψ_f and ψ_τ are positive parameters (shown in the appendix).

Compared to Airaudo et al. (2007), the novel feature of equation (40) is the role of the expectation on the future natural interest rate, $E_t r_{t+1}^n$, which affects the current natural interest rate negatively as a result of the joint effect of the cost channel and the wealth effect. An increase in the expectation of the future natural interest rate, in fact, lowers the future dividends and therefore the asset prices. As a consequence, the natural output gap decreases and this implies a smaller level of the current natural interest rate.

Equation (37) is the New Keynesian Phillips curve augmented by the cost channel. The IS schedule (38) links consumption to the inflation-adjusted return on nominal bonds (i.e. to the real interest rate) and to the asset price dynamics through a wealth effect. Finally, equation (39) describes the dynamics of the real stock price which is influenced by both the supply and the demand shocks.

The natural level of the asset price is given by:

$$q_t^n = y_t^n - \frac{\varsigma}{(1+\psi)} E_t r_{t+1}^n - \frac{(1+\psi-\rho_f)}{1+\psi-\widetilde{\beta}\rho_f} f_t - \frac{(1+\psi_\tau)(1+\psi)(1-\rho_\tau)}{1+\psi-\widetilde{\beta}\rho_\tau} \tau_t \tag{41}$$

where q_t^n is decreasing in the public expenditure shock f_t and in the taste shock τ_t, because their realization implies a reduction in private savings. Note that

[10] All the analytical details can be found in the appendix.

[11] As show by Woodford (2003), if $r_t = r_t^n$, then the output is kept equal to the level that would arise in the absence of nominal rigidities. It follows that the interest rate gap $r_t - r_t^n$ captures the effects on the actual equilibrium due to the presence of nominal rigidities.

an increase in the expectations of the future natural interest rate leads to a decrease in the expectations on the dividends due to a greater marginal costs. Thus, the natural level of the asset price decreases.

Considering the system in log-deviations with respect to the flexible price equilibrium, we obtain:

$$\pi_t = \widetilde{\beta} E_t \pi_{t+1} + \kappa (1+\eta) x_t + \kappa i_t + u_t \tag{42}$$

$$x_t = \frac{1}{1+\psi} E_t x_{t+1} + \frac{\psi}{1+\psi} s_t - \frac{1}{1+\psi} (i_t - E_t \pi_{t+1}) + g_t \tag{43}$$

$$s_t = \widetilde{\beta} E_t s_{t+1} - \delta E_t x_{t+1} - \varsigma E_t i_{t+1} - (i_t - E_t \pi_{t+1}) \tag{44}$$

Substituting (43) in (44) we can rewrite the system as:

$$\pi_t = \widetilde{\beta} E_t \pi_{t+1} + \kappa (1+\eta) x_t + \kappa i_t + u_t \tag{45}$$

$$x_t = \frac{1}{1+\psi} E_t x_{t+1} + \frac{\psi}{1+\psi} s_t - \frac{1}{1+\psi} (i_t - E_t \pi_{t+1}) + g_t \tag{46}$$

$$s_t = \frac{\widetilde{\beta}}{1+\psi} E_t s_{t+1} - \frac{1+\delta}{1+\psi} E_t x_{t+1} + x_t - \frac{\varsigma}{1+\psi} E_t i_{t+1} - g_t \tag{47}$$

where x_t is the output gap, $i_t = r_t - r_t^n$ and $s_t = q_t - q_t^n$ are the log-deviations of the interest rate (respectively the asset price) from the flexible price equilibrium level, $g_t = (1+\psi_\tau) E_t \Delta \tau_{t+1} - \Delta g_{t+1}$ is the demand shock.

The effect of the cost channel on the asset price equation is captured by the presence of the expectation on the interest rate. When the probability to die is zero, $\psi \to 0$ and the model collapses to the standard model of the cost channel with infinitively lived agents.

1.5 Optimal monetary policy under discretion

The central bank sets the interest rate i_t in order to stabilize both inflation and the output gap around the target levels. The optimal monetary policy is obtained by the minimization of a quadratic intertemporal loss function:[12]

$$L = \frac{1}{2} E_t \sum_{s=0}^{\infty} \beta^s \left(\pi_{t+s}^2 + \alpha_x x_{t+s}^2 \right) \tag{48}$$

where the parameter α_x measures the relative importance that the central bank places on output stabilization relative to inflation stabilization.

In a regime of discretion in the standard new Keynesian model, the central bank minimizes the loss function (48) subject only to the New Keynesian Phillips curve. In the presence of the cost channel, since the nominal interest rate

[12] Following a well established tradition, we posit the loss function. Ravenna and Walsh (2006) derive the loss function from "first principle" taking a second order approximation to the utility of the representative agent. It follows that fiscal shocks enter the objective policy function and lead to a policy trade-off between the welfare output gap and inflation, even in the absence of the ad hoc cost push shock.

appears also in equation (45), minimization of equation (48) should be carried out subject to both equations (46) and (45).The optimality condition reads as follows[13]:

$$\pi_t = -\frac{\alpha_x}{(\eta - \psi)\,\kappa} x_t \tag{49}$$

Equation (49) represents the Social Expansion Path (SEP) in our model with cost channel and wealth effect. As one would expect, the output gap and inflation must move in opposite directions. If the probability to die were zero, then ψ would become zero too and we would go back to the SEP derived by Ravenna and Walsh (2007).

Note also that the presence of finitely lived agents affects the social expansion path through the probability of death, γ, which shows up in ψ: the greater is the probability to die, the steeper is the SEP.

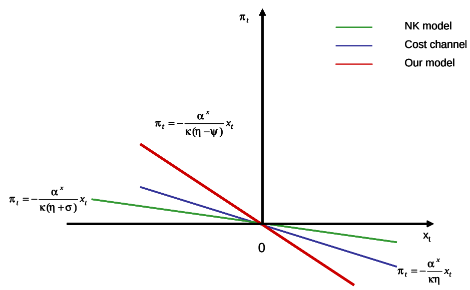

Fig. 3.1: Social Expansion Path

As shown in figure 3.1, in the present model the SEP is steeper than both in the case of the standard new Keynesian model and in the cost channel case.

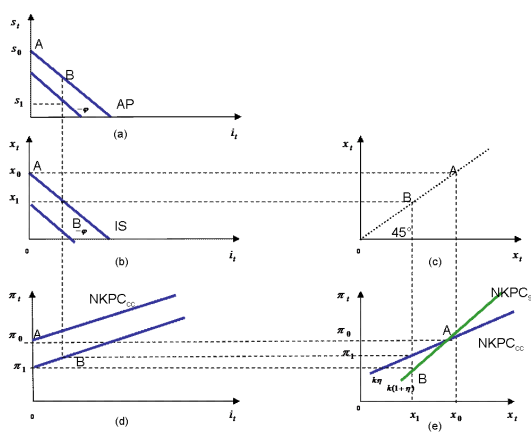

Fig. 3.2: The New Keynesian Phillips Curve

Moreover, the NKPC is flatter. To understand the reason, we consider figure 3.2. Panel (a) and (b) depict the asset price and IS curve as decreasing in the

[13] See the appendix for the details.

interest rate. Initially, the economy is located at the equilibrium (point A): when the interest rate is zero, asset prices and output gap are respectively s_0 and x_0. As the monetary authority raises the interest rate, the economy moves from point A to point B. The asset prices and output gap dynamics depend on the combination of two effects. On one hand,the increase of the interest rate directly leads to a decrease both in asset prices and in output gap. On the other hand, the contraction of asset prices leads to a decrease in output gap (through the wealth effect) and vice versa. With respect to the traditional NKPC$_s$, the curve is flatter.

The net effect is given by the condition:

$$\frac{\partial \pi_t}{\partial i_t} = \kappa - \frac{\kappa (1+\eta)}{1+\psi} = -\frac{\kappa (\eta - \psi)}{1+\psi}$$

The total effect depends on the structural parameters: as $\psi \to 0$, the previous derivative becomes negative and thus the negative effect prevails on the first one, as in the Ravenna and Walsh (2006) model. Note that if $\eta < \psi$, an increase in interest rate determines an increase in the inflation level. Here we consider the case $\eta > \psi$.

Note that the behavior of the central bank in trading off the fluctuations in the output for stabilizing inflation is affected by the cost channel, because stabilizing inflation is now more costly in terms of output gap: repeated changes in the optimal monetary policy increases inflation variability for a given output gap level.

1.5.1 Rational Expectation Equilibrium

Solving jointly equations (46), (45), (47) and (49), we obtain the (pseudo) reduced form:

$$\begin{aligned} y_t &= A + M E_t y_{t+1} + P \varepsilon_t \\ \varepsilon_t &= F \varepsilon_{t-1} + \widetilde{\varepsilon}_t \end{aligned}$$

where $y_t \equiv [\pi_t, x_t, s_t, i_t]'$ and $\varepsilon_t = [g_t, u_t]'$ be the vectors containing the endogenous variables and the fundamental shocks of our economy respectively, $A = [0_4]$, M is the key matrix of structural parameters, P is the matrix of structural parameters of the shocks and F captures the shocks' persistence. All the equations are represented in the appendix.

In the terminology of Evans and Honkapohja, the interest rate i_t assumes the form of an *expectations-based* rule:

$$i_t = \Phi_\pi E_t \pi_{t+1} + \Phi_x E_t x_{t+1} + \Phi_s E_t s_{t+1} + \Phi_i E_t i_{t+1} + \Phi_g g_t + \Phi_u u_t \qquad (50)$$

where

$$
\begin{aligned}
\Phi_\pi &= 1 + \frac{\kappa\left(\widetilde{\beta}+\kappa\right)(\eta-\psi)}{F} \\
\Phi_x &= \frac{\left[\alpha_x + \kappa^2(1+\eta)(\eta-\psi)\right](1-\delta\psi)}{F(1+\psi)} \\
\Phi_s &= \frac{\psi}{1+\psi} + \frac{\left[\alpha_x + \kappa^2(\eta-\psi)\right]\psi}{F}
\end{aligned}
$$

$$
\begin{aligned}
\Phi_i &= -\frac{\xi\left[\alpha_x + (1+\eta)\kappa^2(\eta-\psi)\right]\psi}{F(1+\psi)} \\
\Phi_g &= \frac{(1+\psi)\left[\alpha_x + (1+\eta)\kappa^2(\eta-\psi)\right]}{F} \\
\Phi_u &= -\frac{\kappa(\eta-\psi)}{F}
\end{aligned}
$$

By construction, this rule implements the optimal discretionary policy in every period and for all values of private expectations.

As pointed out previously, our optimal interest rate does not prescribe to offset completely any shocks to the IS curve. The response of the central bank to expected changes in the inflation rate is therefore more aggressive than in the standard model, because a change in the interest rate affects inflation directly through the NKPC and indirectly through the IS schedule.

In Leeper's classification (Leeper, 1991), a monetary policy rule that responds to inflation by raising interest rate less than one-for-one in response to an increase in inflation is said to be *passive* and a rule that directs the central bank to raise the interest rate more than one-for-one is said to be *active*. In our model, the coefficient $\Phi_\pi > 1$ means that central bank adopts an active rule, responding more than one for one to higher inflation's expectations. This result is consistent with Taylor (1993, 1999), Clarida Galì and Gertler (2000) and Woodford (2003). It is said that such rule follows the *Taylor principle*, thanks to which the system should be able to converge to the rational expectation equilibrium.

Conversely, the coefficients Φ_x and Φ_s are less than one and this means that the response to output gap and asset prices is passive. By reacting too aggressively to asset prices the central bank would respond negatively to future output gaps owing to the negative relation between the dividends and the real marginal cost. This result is consistent with the results obtained by Carlstrom and Fuerst (2007) and Airaudo, Nisiticò and Zanna (2007).

Note also that the higher the wealth effect, i.e. the higher ψ, the lower will be the response to inflation's expectations Φ_π and the higher will be the response to asset prices' expectations Φ_s[14]. Conversely, if the wealth effect disappears,

[14] See the appendix for details.

i.e. ψ becomes zero, then our model will go back to the standard New Keynesian model in which asset prices play no an active role, because they do not have any impact on aggregate demand.

The M matrix is as follows:

$$\begin{bmatrix} \frac{\alpha_x\left(\widetilde{\beta}+\kappa\right)}{F} & \frac{\kappa\alpha_x(1-\delta\psi)}{(1+\psi)F} & \frac{\alpha_x\kappa\psi}{(1+\psi)F} & -\frac{\alpha_x\xi\kappa\psi}{(1+\psi)F} \\ -\frac{\kappa\left(\widetilde{\beta}+\kappa\right)(\eta-\psi)}{F} & -\frac{\kappa^2(\eta-\psi)(1-\delta\psi)}{(1+\psi)F} & -\frac{\kappa^2\psi(\eta-\psi)}{(1+\psi)F} & \frac{\psi\xi\kappa^2(\eta-\psi)}{(1+\psi)F} \\ -\frac{\kappa\left(\widetilde{\beta}+\kappa\right)(\eta-\psi)}{F} & -\frac{\alpha_x(1+\delta)+k^2(\eta-\psi)[1+\eta+\delta(\eta-\psi)]}{(1+\psi)F} & \frac{\alpha_x+\kappa^2(\eta-\psi)^2}{(1+\psi)F} & -\frac{\xi\left[\alpha_x+\kappa^2(\eta-\psi)^2\right]}{(1+\psi)F} \\ 1+\frac{\kappa\left(\widetilde{\beta}+\kappa\right)(\eta-\psi)}{F} & \frac{\left[\alpha_x+\kappa^2(1+\eta)(\eta-\psi)\right](1-\delta\psi)}{(1+\psi)F} & \frac{\psi}{1+\psi}+\frac{\left[\alpha_x+\kappa^2(\eta-\psi)\right]\psi}{(1+\psi)F} & -\frac{\xi\left[\alpha_x+(1+\eta)\kappa^2(\eta-\psi)\right]\psi}{(1+\psi)F} \end{bmatrix}$$

where $F = \left[\alpha_x + \kappa^2\eta\left(\eta-\psi\right)\right]$.

Evans and Honkapohja (2001, 2003), McCallum (1983, 1998) and Uhlig (1999) show that if the model is determinate, then an unique REE exists.

The REE can be represented in minimum state variable (MSV) form (McCallum, 2004).

To find the REE, we use the "guess and verify" method. We suppose that a solution takes the form:

$$y_t = a + B\varepsilon_t \tag{51}$$

where a is the slope of our variables and ε_t is a function of both demand and supply shocks g_t and u_t.

According to our guess, the expectations are:

$$E_t y_{t+1} = E_t\left(a + B\varepsilon_{t+1}\right) = a + BF\varepsilon_t \tag{52}$$

Substituting (52) into (51), we obtain:

$$y_t = (A + Ma) + (MBF + P)\,\varepsilon_t \tag{53}$$

which implies that the guess is confirmed if and only if:

$$\begin{aligned} A + Ma &= a \Rightarrow a = 0_4 \\ MBF + P &= B \Rightarrow vec(B) = \left[I - F' \otimes M\right]^{-1} vec(P) \end{aligned}$$

In this way we can rebuild the matrix of the REE coefficients B. We note also that a unique REE exists if and only if $[I - F' \otimes M]$ is invertible.

1.5.2 Instability under the fundamental-based interest rate

Suppose that the central bank knows the true structure of the economy (46), (45), (47) and that it also assumes erroneously that agents have RE in every period. Then, it follows that the optimal interest rule takes the form of a fundamental interest rule of the type

$$i_t = \Phi_u u_t + \Phi_g g_t \tag{54}$$

as defined by Evans and Honkapohja (2003). This rule describes the equilibrium dynamics of the nominal interest rate.

Combining the fundamental rule (54) with (46), (45), (47) and (49), we obtain the reduced form:

$$\begin{bmatrix} x_t \\ \pi_t \\ s_t \end{bmatrix} = \begin{bmatrix} \frac{1-\delta\psi}{1+\psi}-1 & 1 & \frac{\psi}{1+\psi} \\ \frac{(1-\delta\psi)(1+\eta)}{1+\psi} & \widetilde{\beta}+\eta & \frac{\psi(1+\eta)}{1+\psi} \\ -\delta & 1 & 0 \end{bmatrix} \begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \\ E_t s_{t+1} \end{bmatrix} + F(u_t, g_t)$$

As shown in the appendix, we obtain the standard result of the E-instability under learning. Evans and Honkapohja (2003) note that any interest rate rule of the form (54) will result in an E-unstable REE, regardless of its coefficients values Φ_u and Φ_g.

Proposition 1 *Under the fundamental interest rate rule (54), the system is unstable under private agents learning for any structural parameter values, i.e. the economy does not converge to REE.*

As in the traditional New Keynesian model, an increase in $E_t \pi_{t+1}$ leads to an increase in x_t through the IS curve and to an increase in π_t through the NKPC. Over time this drives to upward revisions of expectations and pushes the economy away from the REE. It follows that even if a change in expected inflation is due to factors unrelated to the fundamentals of inflation, it results in a self-fulling increase in π_t.

The instability problem due to the interest rate rule (54) originates from the implicit assumption on the part of policymakers that agents have rational expectations at every point in time.

1.5.3 Determinacy and E-stability under the Expectation based interest rate rule

As argued by Woodford (2003), the presence of indeterminacy is undesirable not only because it allows for non-fundamental shocks but also because it allows for diverging equilibrium paths of inflation, interest rates, output gap and assets in response to fundamentals shocks. Therefore, in this kind of models, we focus on the local analysis of the region of determinacy.

As in Blanchard and Kahn (1980), since none of the three endogenous variables is predetermined, the system (45)-(47) is determinate if and only if all the eigenvalues of the matrix M lie inside the unite circle.

Proposition 2 *Under the optimal monetary policy rule (50) the necessary and sufficient conditions for determinacy is:*

$$0 < \alpha_x < \min \left\{ \frac{\kappa^2 (\eta - \psi) [1 + \eta (1 + \psi)]}{-1 + \widetilde{\beta} + \kappa - \left(1 + \widetilde{\beta}\xi\right) \psi}, \frac{\kappa^2 (\eta - \psi) [(1 + \eta) (1 + \xi) - \delta]}{\kappa + \left(\widetilde{\beta} - 1\right) + \xi \left(\widetilde{\beta} - 1\right)} \right\}$$

Otherwise, the system is indeterminate.

The proof of this proposition is given in the appendix.

Under the conditions (spelled out, for instances, in Marcet and Sargent, 1998 and Evans and Honkapohja, 1999 and 2001),the E-stability governs the local convergence in real time adaptive learning algorithm: if a REE is E-stable in a neighborhood of the equilibrium, then agents are able to learn and therefore to reach the REE.

As shown by Bullard and Mitra (2002), in the case of the forward looking interest rate rule, if the MSV solution is unique, then it must be also E-stable. The converse does not hold, i.e. the E-stability condition does not imply determinacy. This means that when equilibrium is indeterminate, the system may still converge to equilibria that corresponds to MSV solution under some circumstances.

Recently, McCallum (2007) has shown that in a generic purely forward looking model determinacy is a sufficient condition for E-stability if and only if current information is available in the learning process. It follows that if the system is determinate, then it is also E-stable.

To check the E-stability condition, suppose now that agents have not rational expectations at every point in time and therefore they act as econometricians to form their expectations. We assume that they form their expectations by using a recursive learning algorithm, such as recursive least squares, based on the past data produced by the economy. They know the structure of the economy (i.e. the correct linear form) but need to estimate the values of the coefficients from past data. In our case the agents need to estimate an intercept as well as the slope parameter.[15]

Agents have an initial Perceived Law of Motion (PLM) $y_t = \widehat{a} + \widehat{B}\varepsilon_t$[16] and they use it to form their expectations. Note that the information set at time t include the information about the variables that are dated at time t in the model. Therefore we assume that agents have access to y_t and hence can form their expectations as a linear function of $(1, y_t^{'}, \varepsilon_t^{'})'$.

Inserting the PLM into equation (51) we generate an Actual Law of Motion (ALM), i.e. the law of motion of y_t for a given PLM:

$$y_t = (A + M\widehat{a}) + \left(M\widehat{B}F + P\right)\varepsilon_t$$

in which the reduced-form coefficients are time-varying and are function of the structural parameters describing dynamics and of the coefficients representing agents' beliefs.

We can therefore define the T-mapping from the PLM to the ALM in notional time:

$$T(\widehat{a}, \widehat{B}) = \left(A + M\widehat{a}, M\widehat{B}F + F\right)$$

[15] For example, a positive intercept on inflation would signal that agents expect a positive target of $\overline{\pi}$. Since in our model all the targets are zero, we can conclude that the intercept is null.

[16] The notation $\widehat{}$ indicates that agents have not rational expectations.

E- Stability is determined by the matrix differential equation:

$$\frac{d}{d\tau}(\widehat{a},\widehat{B}) = T(\widehat{a},\widehat{B}) - (\widehat{a},\widehat{B})$$

evaluated at the REE solution values $\left(0,\widehat{B}\right)$. The fixed point of this mapping corresponds to the MSV REE of our economy.

Proposition 3 *Under the optimal monetary policy rule (50) the necessary condition for E-stability is that all eigenvalues of the matrix $(F' \otimes M) - I$ have roots with negative real part.*

Given the restrictions imposed on the matrix F it is clear that the determinacy and E-stability conditions exactly coincide. Of course, whether or not the eigenvalues of the matrix M are less than unity depends on the calibration of the structural parameters of the model and the weight assigned to the objective function.

In order to study the dynamics we need to make specific assumption about the structural parameters of the model. As noted by Pfajfar and Santoro (2007), we need to imposed $\eta > \sigma$. Fixing the Frisch elasticity $\eta = 1.5$ and $\sigma = 1$, we can calculate the value for $\kappa = \frac{\lambda}{(\sigma+\eta)}$ from the calibrations proposed by Clarida, Galì and Gertler (2000), McCallum and Nelson (1999), Woodford (1999) and Walsh and Ravenna (2006):

	κ
CGG	0.0375
MCN	0.15
W	0.012
RW	0.0858

Following Nisticò (2005), we set the elasticity of substitution among intermediate goods θ_t at 21. In this case we obtain a steady state net markup rate of 5% .Considering the discount factor at $\beta = 0.99$ and the net steady state quarterly interest rate $r_t = 0.01$,we can calculate endogenously the parameters δ, ξ and $\widetilde{\beta} = \frac{\beta}{1+\psi}$ where $\psi = \beta(1+r) - 1$.

We can conclude that the REE is determinate and E-stable for all our model's calibrations and the bigger is the parameter κ, the bigger is the value that α_x can assume.

1.6 When asset price targeting matters

In this section we modify the loss function of the policymaker in order to include explicitly a "financial stability target". Therefore in this case the central bank aims at stabilizing inflation, the output gap and stock market dynamics:

$$L = \frac{1}{2}E_t\sum_{s=0}^{\infty}\beta^s\left(\pi_{t+s}^2 + \alpha_x x_{t+s}^2 + \alpha_s s_{t+s}^2\right) \tag{55}$$

where the parameter α_x measures the relative importance of output and α_s is the relative importance of stock prices dynamics that that captures explicitly the financial stability target.

As usual, the central bank observes the fundamentals and the shocks occurred in the economy and modifies the nominal interest rate i_t in order to satisfy its policy objective.

Since the nominal interest rate appears in all the equations that describe our economy, the central bank's problem consists in the minimization of equation (**??**) under equations (46), (45) and (47).

The optimal condition is:

$$\pi_t = -\frac{\alpha_x x_t + \alpha_s s_t}{\eta\kappa} \tag{56}$$

Output gap moves again in the opposite direction of inflation and asset prices misalignments. Note that the SEP does not depend on the probability γ.

1.6.1 Rational Expectation Equilibrium

Solving jointly equations (46), (45), (47) and (56), we obtain the (pseudo) reduced form that can be summed up as:

$$\begin{aligned} y_t &= A + NE_t y_{t+1} + P\varepsilon_t \\ \varepsilon_t &= F\varepsilon_{t-1} + \widetilde{\varepsilon}_t \end{aligned}$$

where $y_t \equiv [\pi_t, x_t, s_t, i_t]'$ and $A = [0_4]$, $\varepsilon_t = [g_t, u_t]'$, N and P are the structural matrices of parameters and F is the matrix of shocks' persistence.

Since we are interested in the implications for determinacy and E-stability of the optimal monetary policy under RE, we find the solution at equilibrium using the undetermined coefficients. Therefore we guess that the solution takes the form of:

$$y_t = a + B\varepsilon_t$$

where a is the slope of our variables and ε_t is a function of both demand g_t and supply shocks u_t. According to our guess, the expectations are given by:

$$E_t y_{t+1} = a + BF\varepsilon_t \tag{57}$$

Inserting (52) into the (pseudo) reduced form (**??**), we obtain:

$$y_t = (A + Na) + (NBF + P)\,\varepsilon_t \tag{58}$$

which implies that guess is confirmed if and only if:

$$\begin{aligned} A + Na &= a \Rightarrow a = 0_4 \\ NBF + P &= B \Rightarrow vec(B) = \left[I - F' \otimes N\right]^{-1} vec(P) \end{aligned}$$

1.6.2 Determinacy and E-stability under the expectation based rule

The central bank sets the interest rate according to an expectation-based rule[17]:

$$
\begin{aligned}
i_t \quad = \quad & \left[1+\frac{\eta\kappa\left(\widetilde{\beta}+\kappa\right)}{\Theta}\right]E_t\pi_{t+1}+\frac{\left[\alpha_x+\eta\kappa^2\left(1+\eta\right)\right]\left(1-\delta\psi\right)-\delta\alpha_s}{\Theta\left(1+\psi\right)}E_tx_{t+1}+ \\
& +\left[\frac{\psi}{1+\psi}+\frac{\alpha_s+\psi\eta\kappa^2}{\Theta\left(1+\psi\right)}\right]E_ts_{t+1}-\xi\left[\frac{\psi}{\left(1+\psi\right)}+\frac{\left(\alpha_s+\eta\kappa^2\psi\right)}{\Theta\left(1+\psi\right)}\right]E_ti_{t+1} \\
& +\frac{\alpha_x+\alpha_s+\eta\kappa^2\left(1+\eta\right)\left(1+\psi\right)}{\Theta}g_t+\frac{\eta\kappa}{\Theta}u_t
\end{aligned}
$$

The key matrix of the reduced form is N:

$$
\begin{bmatrix}
\frac{(\alpha_x+\alpha_s)\left(\widetilde{\beta}+\kappa\right)}{\Theta} & \frac{\kappa[(\alpha_x+\alpha_s)(1-\delta\psi)-\alpha_s\eta\psi(1+\delta)]}{\Theta(1+\psi)} & \frac{\kappa[(\alpha_x+\alpha_s)\psi-\eta\alpha_s]}{\Theta(1+\psi)} & -\frac{\alpha_x\xi\kappa\psi}{(1+\psi)F} \\
\frac{\widetilde{\beta}+\kappa}{\Theta} & \frac{\alpha_s(1+\delta)+\kappa^2\eta(\delta\psi-1)}{\Theta(1+\psi)} & -\frac{\alpha_s+\eta\kappa\psi}{\Theta(1+\psi)} & \frac{\kappa\xi[\eta\alpha_s-(\alpha_x+\alpha_s)\psi]}{\Theta(1+\psi)} \\
\frac{\widetilde{\beta}+\kappa}{\Theta} & -\frac{\alpha_x(1+\delta)+\eta\kappa^2[1+\eta+\delta(\eta-\psi)]}{\Theta(1+\psi)} & \frac{\alpha_x+\kappa^2\eta(\eta-\psi)}{\Theta(1+\psi)} & \frac{\xi\left[\alpha_x+\kappa^2\eta(\psi-\eta)\right]}{\Theta(1+\psi)} \\
1+\frac{\eta\kappa\left(\widetilde{\beta}+\kappa\right)}{\Theta} & \frac{\left[\alpha_x+\eta\kappa^2(1+\eta)\right](1-\delta\psi)-\delta\alpha_s}{\Theta(1+\psi)} & \frac{\psi}{(1+\psi)}+\frac{\alpha_s+\psi\eta\kappa^2}{\Theta(1+\psi)} & -\xi\left[\frac{\psi}{(1+\psi)}+\frac{\left(\alpha_s+\eta\kappa^2\psi\right)}{\Theta(1+\psi)}\right]
\end{bmatrix}
$$

where $\Theta=\left[\alpha_s+\alpha_x+\kappa^2\eta\left(\eta-\psi\right)\right].$

Since it is hard to derive clear analytical results, we present a numerical simulation on a calibrated version of our economy and check the determinacy area. For each pair (α_x,α_s) our numerical routine checks the eigenvalues of the matrix N to determine whether all the eigenvalues have real part less than unity. Regions where the REE is determinate (and thus E-stable) are shown in dark grey. Regions where at least one eigenvalue have a real part greater than unity are grey, i.e. the REE is indeterminate.

Trying to understand the implications of equity price movements' target for the optimal monetary policy, we consider firstly the case of a standard infinitively lived representative agent, i.e. $\gamma=0$.

Figure 3.3 shows that the determinacy area seems to depend on the relative weight assigned to stock prices fluctuations α_s and output gap α_x. In particular, determinacy (and thus E-stability) is obtained for very small values of α_x and α_s.

[17] Note that if the central bank sets the interest rate rule to respond to all the shocks hitting the economy, such as $i_t=\Phi_u u_t+\Phi_g g_t$, we know that the system is both indeterminate and E-unstable.

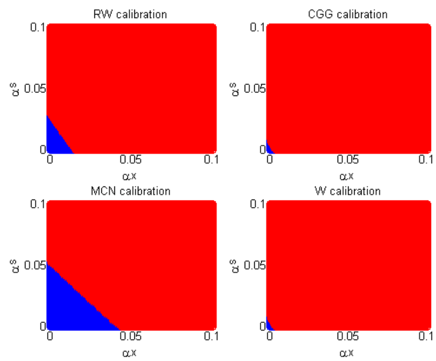

Fig. 3.3: Determinacy area if there is no wealth effect.

Unlike the Evans and Honkapohja (2003)'s results, the expectations-based rule does not induce E-stability for large area of parameters' space in the case of infinitely lived agents. Since the calibration differs just in the value of the key parameter κ, we note that the bigger is κ, the wider is the determinacy area. This is consistent with the results obtained by Airaudo et al. (2007).

The intuition could be the following. For a fixed value of α_x, as the reaction to asset price misalignments α_s increases, the response to expected inflation $\Phi_\pi = 1 + \frac{\eta\kappa\left(\widetilde{\beta}+\kappa\right)}{\Theta} \to 1$ and expected output gap $\Phi_x = \frac{\left[\alpha_x+\eta\kappa^2(1+\eta)\right]-\delta\alpha_s}{\Theta}$ become smaller, while the response to expected asset prices $\Phi_s = \frac{\alpha_s}{\Theta}$ goes up. Since the Taylor principle fails, indeterminacy follows.

To sum up, if agents are infinitively lived, including equity prices in the central bank loss function proves self-defeating to achieve determinacy in an optimal monetary policy under discretion. This conclusion is not true anymore when the wealth effect is taken into account.

Figure 3.4 shows the determinacy area when the agents have a finite horizon:

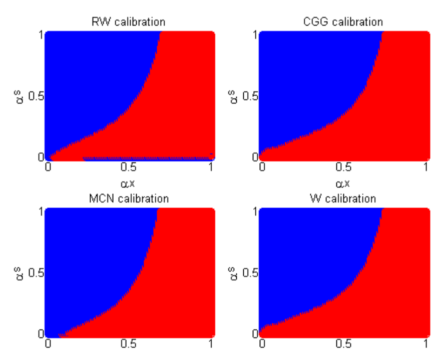

Fig. 3.4: Determinacy area when $\gamma > 0$.

For Ravenna and Walsh's calibration, the determinacy area includes the horizontal axis. If this area was the only one, we could conclude that if the central bank puts a sufficient weight on output gap ceteris paribus, financial stability is essentially irrelevant to achieve the economic stability.

Nevertheless, for all calibrations the wide region of determinacy includes the financial stabilization. In particular, as mentioned above, changes in the interest rate affect the economy in two ways in addition to the supply side. First, it generates a fall in output gap due to a direct effect on IS schedule. Secondly, the interest rate indirectly acts on IS through the misalignments in asset prices. This effect generates a further fall in output gap and a reduction in inflation. By the definition of asset prices, the contraction in aggregate demand simultaneously generates a jump downwards of stock prices. Therefore, financial stability makes less relevant stabilizing output gap.

However, the wider determinacy area implies a positive response both to output gap and asset prices misalignments. Therefore, we can conclude that in the presence of a cost channel, we can find some pairs (α_x, α_s) for which responding to asset prices fluctuations is an optimal policy. This result contrasts to that obtained by a New Keynesian model by Airaudo et al. (2007).

1.7 Conclusion

The aim of this paper is to study whether the central bank should concern about the stock market dynamics in the design of their optimal monetary policy under discretion, when the cost channel matters.

Developing a New Keynesian version with cost channel of Blanchard (1985) and Yaari (1965), we consider an economy where asset prices affect the IS schedule through a wealth effect, as a consequence of the finite planning horizon of the agents.

We show that according to the evidence, the central bank should pay attention to asset prices' s movements when it decides the monetary policy.

From the determinacy and learning analysis we demonstrate that to obtain a unique and E-stable equilibrium, the optimal interest rule should satisfy the Taylor principle. Note that as the wealth effect becomes stronger, the optimal response to asset prices should increase.

When monetary authority cares about output gap, inflation and financial stability, we find that REE is indeterminate for large parameters' region and thus E-unstable, if the agents live infinitely. In this case, the stock price targeting is destabilizing, in accordance with the policy prescriptions derived within the standard New Keynesian model. Contrary to the traditional view, when wealth effect becomes relevant, central bank should be concerned for stabilizing stock price fluctuations.

1.8 Appendix

1.8.1 Model setup

Aggregation through cohorts We follow the methodology implied by Nisticò (2005). Considering equation (8) and (6), the budget constraint holds with equality:

$$P_t C_{s,t}(j) + E_t \left\{ F^s_{t,t+1} (1-\gamma) A_{s,t+1}(j) \right\} = W_t N_{s,t}(j) - P_t T_{s,t}(j) + A_{s,t}(j) \quad (59)$$

Solving forward and substituting the definition of human wealth (11), we obtain:

$$\begin{aligned} A_{s,t}(j) &= \underbrace{P_t C_{s,t}(j) + E_t \left\{ F^s_{t,t+1} (1-\gamma) A_{s,t+1}(j) \right\}}_{E_t \sum_{k=0}^{\infty} F^s_{t,t+1} (1-\gamma)^k P_{t+k} C_{t+k}} - \underbrace{\left[W_t N_{s,t}(j) - P_t T_{s,t}(j) \right]}_{h_t(j)} \\ &= E_t \sum_{k=0}^{\infty} F^s_{t,t+1} (1-\gamma)^k P_{t+k} C_{t+k} - h_t(j) \end{aligned} \quad (60)$$

From equation (4) we can obtain the equilibrium stochastic discount factor for k period ahead:

$$F^s_{t,t+k} = \beta \frac{P_t C_{s,t}(j) \tau_{t+1}}{P_{t+1} C_{s,t+1}(j) \tau_t} = \prod_{i=0}^{k-1} F^s_{t+i,t+i-1} \quad (61)$$

and substituting the previous equation into (60), we obtain

$$A_{s,t}(j) = P_t C_{s,t} \underbrace{E_t \sum_{k=0}^{\infty} F^s_{t,t+1} (1-\gamma)^k \frac{\tau_{t+1}}{\tau_t}}_{\Omega_t} - h_t(j) = \Omega_t P_t C_{s,t} - h_t(j)$$

therefore

$$P_t C_{s,t} = \frac{1}{\Omega_t} \left[A_{s,t}(j) + h_t(j) \right] \quad (62)$$

In order to aggregate the consumption function, consider the definition of human wealth (11):

$$\begin{aligned} h_t(j) &\equiv E_t \left\{ \sum_{k=0}^{\infty} F^s_{t,t+k} (1-\gamma)^k \left[W_{t+k} N_{t+k}(j) - P_{t+k} T_{t+k}(j) \right] \right\} \\ &= \left[W_t N_t(j) - P_t T_t(j) \right] + E_t \left\{ F^s_{t,t+1} (1-\gamma) h_{t+1}(j) \right\} \end{aligned} \quad (63)$$

Leading equation (62) forward one period, we get:

$$\begin{aligned} P_t C_{s,t} &= \frac{1}{\Omega_t} \left[A_{s,t}(j) + h_t(j) \right] \\ E_t \left\{ F^s_{t,t+1} (1-\gamma) h_{t+1}(j) \right\} &= E_t \left\{ F^s_{t,t+1} (1-\gamma) \Omega_{t+1} P_{t+1} C_{s,t+1} \right\} + \\ &\quad - E_t \left\{ F^s_{t,t+1} (1-\gamma) A_{s,t+1}(j) \right\} \end{aligned} \quad (64)$$

and inserting it in the equation (63) we obtain

$$\begin{aligned}
h_t(j) &= [W_t N_t(j) - P_t T_t(j)] + E_t\left\{\Gamma^{s}_{t,t+1}(1-\gamma)\, h_{t+1}(j)\right\} \\
&= [W_t N_t(j) - P_t T_t(j)] + E_t\left\{\Gamma^{s}_{t,t+1}(1-\gamma)\,\Omega_{t+1}P_{t+1}C_{s,t+1}\right\} \\
&\quad -E_t\left\{\Gamma^{s}_{t,t+1}(1-\gamma)\, A_{s,t+1}(j)\right\}
\end{aligned} \tag{65}$$

Finally, replacing in equation (62) $h_t(j)$ with (65) and $A_{s,t}(j)$ with (59) we find:

$$\begin{aligned}
P_t C_{s,t} &= \frac{1}{\Omega_t}\left[A_{s,t}(j) + h_t(j)\right] \\
(\Omega_t - 1) P_t C_{s,t} &= E_t\left\{\Gamma^{s}_{t,t+1}(1-\gamma)\,\Omega_{t+1}P_{t+1}C_{s,t+1}\right\} + \gamma E_t\left\{\Gamma^{s}_{t,t+1}A_{s,t+1}(j)\right\}
\end{aligned} \tag{66}$$

Wholesale Sector Under the monopolistic competition the intermediate sector firm j maximizes their profits subject the time of price adjustment:

$$\underset{P_t(j)}{Max} E_t\left\{\sum_{i=0}^{\infty}\omega^i F_{i,t+i} Y_t(j)\left[\frac{P_t(j)}{P_{t+i}} - \varphi_{t+i}\right]\right\}$$

Note that ω^i indicates the probability that the price $P_t(j)$ is adjusted, $F_{t,t+i}$ is the discount factor and φ_{t+i} is the real marginal cost (20). Substituting the demand for intermediate goods (17), the previous equation becomes:

$$\underset{P_t(j)}{Max} E_t\left\{\sum_{i=0}^{\infty}\omega^i F_{i,t+i} Y_t P_{t+i}^{\theta_t}\left[P_t(j)^{1-\theta_t} - \varphi_{t+i}P_t(j)^{-\theta_t}\right]\right\}$$

The first order condition implies that all firms revising their prices at time t will choose a common optimal price level P_t^*:

$$E_t\left\{\sum_{i=0}^{\infty}\omega^i F_{i,t+i} Y_t P_t^{\theta_t}\left[(1-\theta_t)\, P_t(j)^{-\theta_t} + \theta_t\varphi_{t+i}P_t(j)^{-1-\theta_t}\right]\right\} = 0 \tag{67}$$

Multiplying (67) by $P_t(j)$ and dividing by $1-\theta_t$, we obtain:

$$E_t\left\{\sum_{i=0}^{\infty}\omega^i F_{i,t+i} Y_t P_{t+i}^{-\theta_t}\left[\frac{(1-\theta_t)}{(1-\theta_t)}P_t(j) - \underbrace{\frac{\theta_t}{(1-\theta_t)}}_{\Phi}\varphi_{t+i}\right]\right\} = 0 \tag{68}$$

$$E_t\left\{\sum_{i=0}^{\infty}\omega^i F_{i,t+i} Y_t P_{t+i}^{-\theta_t}\left[P_t(j) - \Phi\varphi_{t+i}\right]\right\} = 0 \tag{69}$$

where Φ is the gross markup. Remind that $F_{t,t+1} = \widetilde{\beta}$ and choose a symmetric equilibrium the optimal relative price p_t^*:

$$p_t^* \equiv \log\left(\frac{P_t^*}{P}\right) = \left(1-\omega\widetilde{\beta}\right) E_t \left\{\sum_{i=0}^{\infty}\left(\omega\widetilde{\beta}\right)^i \left(\varphi_{t+i} + p_{t+i}\right) - (\Phi - 1)\log\left(\frac{\theta_t}{\theta}\right)\right\}$$

In the case of *flexible prices ($\omega = 0$)* and in *absence* of any *inefficient shocks to* the *markup*, in every period the prices are adjusted by firms. As a consequence the firms set their price as a constant markup over the nominal marginal cost:

$$p_t^* = \Phi\varphi_t^n P_t$$

where the real marginal cost is defined as $\widehat{\varphi}_t^n$:

$$\varphi_t^n \equiv \log\varphi_t^n - \log\varphi_t = 0$$

Otherwise, in the case of *sticky prices ($\omega > 0$)*, firms take into account the opportunity cost of adjusting their prices. The average price is therefore:

$$p_t^* = \omega\widetilde{\beta}E_t p_{t+1}^* + \left(1-\omega\widetilde{\beta}\right)\left(\widehat{\varphi}_t + p_t\right) - (\Phi - 1)\log\left(\frac{\theta_t}{\theta}\right) \tag{70}$$

Since the average price of nonadjuster is just the average price of all those firms in the period $t-1$, the steady state equilibrium leads to:

$$\pi_t = \widetilde{\beta}E_t\pi_{t+1} + \kappa\varphi_t + u_t \tag{71}$$

where $u_t = (1-\Phi)\frac{(1-\omega)(1-\omega\beta)}{\omega}\ln\frac{\theta_t}{\theta}$ is a cost push shock.

From the definition of real marginal cost (35), the productivity function (31), the economy's constraint (29) and the definition of natural level of output (36), we obtain:

$$\begin{aligned}\varphi_t &= (1+\eta)\left[y_t - \underbrace{z_t - \frac{1}{(1+\eta)}\left(\tau_t + \varkappa + f_t\right) + \frac{1}{(1+\eta)}r_t}_{y_t^n} - \frac{1}{(1+\eta)}r_t + \frac{1}{(1+\eta)}r_t^n\right] \\ &= (1+\eta)\,x_t + i_t \end{aligned} \tag{72}$$

Let define the output gap $x_t = \widehat{y}_t - \widehat{y}_t^f$ and $i_t = r_t - r_t^n$, the equation (71) becomes:

$$\pi_t = \widetilde{\beta}E_t\pi_{t+1} + \kappa(1+\eta)\,x_t + \kappa i_t + u_t$$

1.8.2 Steady State

In the long run the propensity to consume out financial wealth is not affect by the taste shocks:

$$\begin{aligned}\frac{1}{\Omega_t} &= \frac{1}{\sum_{k=0}^{\infty}\beta^k(1-\gamma)^k}\frac{\tau_{t+1}}{\tau_t} \\ &= 1 - \beta(1-\gamma)\end{aligned} \tag{73}$$

It follows that the log-linearization of Ω_t is:

$$\begin{aligned}\Omega_t &\equiv \log\left(\frac{\Omega_t}{\Omega}\right) \\ &= \frac{\beta(1-\gamma)}{1-\beta(1-\gamma)\rho_\tau}E_t\Delta\tau_{t+1} \qquad (74)\end{aligned}$$

The market clearing condition implies that $Y_t = C_t + F_t$. Assuming that the government expenditure is a stochastic fraction of the current output $F_t = \varrho Y_t$ and this is totally financed by lump sum taxation, in order to have a balanced budget constraint fiscal policy, i.e. $F_t = T_t$.

The Log-linearization of the market clearing condition is:

$$y_t = c_t + f_t \qquad (75)$$

where $f_t = -\log\frac{1-\varrho_t}{1-\varrho_t}$ and we assume that ϱ_t follows an autoregressive stochastic process, i.e. $f_t = \rho_f f_{t-1} + \widetilde{\varepsilon}_{f,t}$ with $0 < \rho_f < 1$ and $\widetilde{\varepsilon}_{f,t} \sim N\left(0, \sigma_f^2\right)$.

From equation (66) and from the definition of Ω_t (73) a zero inflation steady state defines the following condition:

$$\begin{aligned}\beta(1+r_t) &= \underbrace{\gamma\frac{1-\beta(1-\gamma)}{(1-\gamma)}\frac{A_t}{P_tC_t}}_{\psi}+1 \\ &= \psi+1\end{aligned}$$

Note that $\frac{\partial\psi}{\partial\gamma} = \left[-\beta + \frac{1}{(\gamma-1)^2}\right]\frac{A_t}{P_tC_t} > 0$.

Given the values of the structural parameters β, γ and Φ and the steady state share of consumption $\frac{C_t}{Y_t}$ we can calculate the steady state level of the real wealth to consumption $\frac{A_t}{P_tC_t}$, the net interest rate r_t and the parameter ψ.

In particular, considering the resource constraint $Y_t = C_t + F_t$ where $F_t = \varrho Y_t$, the production function $Y_t = Z_tN_t$, the steady state marginal cost is equal to the inverse of the markup Φ and the marginal cost $\varphi_t = \frac{W_tR_t}{P_tZ_t}$, we get that the steady state value of the hours worked is $N = \frac{1}{1+R\Phi(1-\varrho)}$. It follows that the steady state Frisch elasticity of the labor supply is $\eta = \frac{N}{1-N} = \frac{1}{R\Phi(1-\varrho)}$.

In absence of a stochastic shocks, the production function, the equilibrium on the labor market and equation (23) define the steady state equilibria values for:

$$\begin{aligned}\beta(1+r_t) &= 1+\gamma\frac{1-\beta(1-\gamma)}{(1-\gamma)}\frac{A_t}{P_tC_t} \qquad (76)\\ A_t &= P_t(Q_t+D_t) \qquad (77)\\ Q_t+D_t &= (1+r_t)Q_t \qquad (78)\end{aligned}$$

$$
\begin{aligned}
Y_t &= A_t N_t && (79)\\
\varphi_t &= \frac{W_t R_t}{P_t Z_t} = \frac{1}{\Phi} && (80)\\
D_t &= Y_t - \frac{W_t R_t}{P_t Z_t} Y_t = Y_t - \varphi_t Y_t = \frac{\Phi - 1}{\Phi} Y_t && (81)
\end{aligned}
$$

Combining equations (77), (78) and (81) with the steady state expression for the consumption, we have:

$$
\frac{A_t}{P_t C_t} = \frac{\frac{\Phi-1}{\Phi}}{1-\varrho} \frac{1+r_t}{r_t}
$$

Therefore, equation (76) gives us the possibility to rewrite the steady state real interest rate r_t as a function of some structural parameters of the model:

$$
\beta\left(1+r_t\right) = 1 + \gamma \frac{1-\beta\left(1-\gamma\right)}{\left(1-\gamma\right)} \frac{\frac{\Phi-1}{\Phi}}{1-\varrho} \frac{1+r_t}{r_t} \qquad (82)
$$

From the definition of the ψ and equation (82), we obtain $\psi = \beta\left(1+r\right) - 1$ or $\widetilde{\beta}\left(1+r\right) = 1$. Note that ψ is strictly increasing in both γ and Φ, as in Airaudo et al. (2007):

$$
\begin{aligned}
\frac{\partial \psi}{\partial \gamma} &= \frac{\left(1+r_t\right)\left(\Phi-1\right)\left[\beta\left(\gamma-1\right)^2 - 1\right]}{\Phi r_t \left(\gamma-1\right)^2 \left(\varrho-1\right)} > 0\\
\frac{\partial \psi}{\partial \Phi} &= \frac{\gamma\left(1+r_t\right)\left[1+\beta\left(\gamma-1\right)\right]}{\Phi^2 r_t \left(\gamma-1\right)\left(\varrho-1\right)} > 0
\end{aligned}
$$

The log-linear approximation of the Euler equation for consumption (25) becomes:

$$
\begin{aligned}
c_t = {} & \frac{\left(1-\gamma\right)\Omega_t}{\left(\Omega_t-1\right)\left(1+r_t\right)}\left[E_t c_{t+1} - \left(r_t - \widetilde{\rho}\right) + E_t \pi_{t+1}\right] + \frac{\gamma \frac{A_t}{P_t C_t}}{\left(\Omega_t-1\right)\left(1+r_t\right)} q_t + \\
& - \frac{\Omega_t}{\Omega_t - 1}\left[1 - \frac{\left(1-\gamma\right)}{\left(1+r_t\right)}\rho_\tau\right]\sigma_t
\end{aligned}
$$

where $\psi_\tau = \frac{\psi[\beta(1-\gamma)\rho_\tau]}{(1+\psi)[1-\beta(1-\gamma)\rho_\tau]}$ and $\frac{A_t}{P_t C_t} = \frac{\Phi-1}{\Phi}\frac{1}{1-\varrho}\frac{1+r_t}{r_t}$.

After some algebra, the linear approximation of the pricing equation (6) gives:

$$
q_t = \frac{1}{1+r_t} E_t q_{t+1} + \underbrace{\frac{D_t}{\left(1+r\right)Q_t}}_{\frac{r_t}{1+r_t}} E_t d_{t+1} - \left(r_t - E_t \pi_{t+1} + \widetilde{\rho}\right)
$$

Substituting the definition of $\widetilde{\beta}$ and using equation (78) we obtain:

$$
q_t = \widetilde{\beta} E_t q_{t+1} + \left(1 - \widetilde{\beta}\right) E_t d_{t+1} - \left(r_t - E_t \pi_{t+1} + \widetilde{\rho}\right)
$$

Log-linearinzing equation (23) considering the linear production function and (31) the marginal cost (35), we obtain:

$$
\begin{aligned}
d_t &= \frac{Y}{D}y_t - \frac{WNR}{PD}\left(n_t + w_t - p_t + r_t\right) \\
&= \frac{Y}{D}y_t - \frac{WNR}{PD}\left(y_t - \underbrace{z_t + r_t + w_t - p_t}_{\varphi_t}\right) \\
&= \underbrace{\frac{Y}{D}y_t - \frac{WNR}{PD}y_t}_{\frac{y}{D}\left(Y - \frac{WNR}{P}\right) = \frac{y_t}{D}D} - \frac{W_t N_t R_t}{P_t D_t}\varphi_t \\
&= y_t - \frac{WNR_t}{PD}\varphi_t
\end{aligned}
$$

Note that rearranging the expression $\frac{W_t N_t R_t}{P_t D_t}$ we obtain:

$$
\frac{WNR}{PD} = \underbrace{\frac{RW\frac{Y}{Z}}{PD}}_{\varphi = \frac{WR}{ZP}} = \underbrace{\frac{\varphi Y}{D}}_{\frac{1}{D} = \frac{\widetilde{\beta}}{1-\widetilde{\beta}}\frac{1}{Q}} = \frac{\widetilde{\beta}}{1-\widetilde{\beta}}\frac{Y}{Q}\frac{1}{\Phi}
$$

and using the definition of marginal cost (72), adding and subtracting y_t^n we get:

$$
\begin{aligned}
d_t &= y_t - \frac{\widetilde{\beta}}{1-\widetilde{\beta}}\frac{Y}{Q}\frac{1}{\Phi}\varphi \\
&= y_t^n - \frac{\widetilde{\beta}\left[(1+\eta)\right]\frac{Y}{Q} - \Phi\left(1-\widetilde{\beta}\right)}{\Phi\left(1-\widetilde{\beta}\right)}x_t - \frac{\widetilde{\beta}\frac{Y}{Q}}{\Phi\left(1-\widetilde{\beta}\right)}r_t \qquad (83)
\end{aligned}
$$

Note that dividends and output are negatively correlated and its magnitude depends on the steady state markup Φ.

Inserting the previous equation (83) into (33), we have:

$$
\begin{aligned}
q_t &= \widetilde{\beta}E_t q_{t+1} + \left(1-\widetilde{\beta}\right)E_t d_{t+1} - \left(r_t - E_t\pi_{t+1} - \widetilde{\rho}\right) \\
&= \widetilde{\beta}E_t q_{t+1} + \left(1-\widetilde{\beta}\right)E_t y_{t+1}^n - \underbrace{\frac{\widetilde{\beta}\left[(1+\eta)\right]\frac{Y_t}{Q_t} - \left(1-\widetilde{\beta}\right)}{\Phi}}_{\delta}E_t x_{t+1} - \underbrace{\frac{\widetilde{\beta}\frac{Y_t}{Q_t}}{\Phi}}_{\varsigma}E_t r_{t+1} + \\
&\quad - \left(r_t - E_t\pi_{t+1} - \widetilde{\rho}\right)
\end{aligned}
$$

Note that combining equation (78), (81) with $\widetilde{\beta} = \frac{1}{1+r_t} = \frac{\beta}{1+\psi}$, we obtain that $\frac{Y_t}{Q_t} = \frac{r_t \Phi}{\Phi - 1}$.

$$\begin{aligned} \delta &= \left(\frac{1+\eta}{\Phi - 1} - 1 \right) \frac{r_t}{1 + r_t} \\ \varsigma &= \left(\frac{r_t}{1 + r_t} \right) \left(\frac{1}{\Phi - 1} \right) \end{aligned}$$

Note that $\frac{\partial \delta}{\partial \Phi} = -\frac{1+\eta}{\Phi^2} < 0$, i.e. Φ is strictly decreasing in Φ and $\frac{\partial \delta}{\partial \gamma} = \left(\frac{1+\eta}{\Phi - 1} - 1 \right) \frac{1}{(1+r)^2} \frac{\partial r}{\partial \gamma} > 0$ means that δ is increasing in γ. Similarly, $\frac{\partial v}{\partial \Phi} = -\frac{1}{\Phi^2} < 0$ and $\frac{\partial \delta}{\partial \gamma} = \left(\frac{1}{\Phi - 1} \right) \frac{1}{(1+r)^2} \frac{\partial r}{\partial \gamma} > 0$.

1.8.3 Natural interest rate

The *Wicksellian real interest rate* $\widetilde{r}_t^n$ is obtained by the two equations system:

$$\begin{aligned} y_t^n &= E_t^n y_{t+1} + \psi \left(q_t^n - y_t^n \right) - \left(\widetilde{r}_t^n - \widetilde{\rho} \right) - (1+\psi)(1+\psi_\tau) E_t \tau_{t+1} + \left(1 + \psi - \rho_f \right) f_t \quad (84) \\ q_t^n &= \widetilde{\beta} E_t q_{t+1}^n + \left(1 - \widetilde{\beta} \right) E_t y_{t+1}^n - \varsigma E_t r_{t+1}^n - \left(\widetilde{r}_t^n - \widetilde{\rho} \right) \quad (85) \end{aligned}$$

If we eliminate the wealth effect (i.e. $\gamma = \psi = \psi_\tau = 0$), we can obtain the natural interest rate of the New Keynesian model with cost channel:

$$\widetilde{rr}_t^n = \rho + E_t \Delta a_{t+1} + \frac{1}{1+\eta} \left(E_t \Delta \varkappa_{t+1} - E_t \Delta r_{t+1} \right) - \frac{\eta}{1+\eta} \left(E_t \Delta \tau_{t+1} + E_t \Delta g_{t+1} \right) \quad (86)$$

From equation (84) we solve for $\left(\widetilde{r}_t^n - \widetilde{\rho} \right)$ and than we put the result into equation (85):

$$\left(\widetilde{r}_t^n - \widetilde{\rho} \right) = \underbrace{\left(E y_{t+1}^n - y_t^n \right)}_{E_t \Delta y_{t+1}^n} + \psi \left(q_t^n - y_t^n \right) - (1+\psi)(1+\psi_\tau) E_t \tau_{t+1} + \underbrace{\left(1 + \psi - \rho_f \right) f_t}_{\psi f_t + \left(1 - \rho_f \right) f_t = \psi f_t - E_t \Delta f_{t+1}} \quad (87)$$

$$\begin{aligned} q_t^n - y_t^n &= \frac{\widetilde{\beta}}{(1+\psi)} \left(E_t q_{t+1}^n - E_t y_{t+1}^n \right) - \frac{\varsigma}{(1+\psi)} E_t r_{t+1}^n + \frac{1}{(1+\psi)} E_t \Delta g_{t+1} + \quad (88) \\ &+ (1+\psi_\tau) E_t \Delta \tau_{t+1} - \frac{\psi}{(1+\psi)} f_t \end{aligned}$$

Considering the rule $E_t \Delta v_{t+k+1} = \rho_\tau E_t v_{t+1}$, $0 < \frac{\widetilde{\beta}}{1+\psi} < 1$, $0 < \rho_\tau < 1$ and iterating forward the equation (88), we get:

$$\begin{aligned}
q_t^n - y_t^n + \frac{\varsigma}{(1+\psi)} E_t r_{t+1}^n &= \lim_{k\to\infty} \left\{ \left(\frac{\widetilde{\beta}}{1+\psi} \right)^k \left(E_t q_{t+1}^n - E_t y_{t+1}^n \right) \right\} + \\
&\quad + \left[\frac{1}{1+\psi} \sum_{k=0}^{\infty} \left(\frac{\widetilde{\beta}\rho_f}{1+\psi} \right) \right]^k \left(E_t \Delta g_{t+1} - \psi g_t \right) + \\
&\quad + \left[(1+\psi_\tau) \sum_{k=0}^{\infty} \left(\frac{\widetilde{\beta}\rho_\tau}{1+\psi} \right) \right]^k E_t \Delta \tau_{t+1} \\
&= \frac{1}{1+\psi-\widetilde{\beta}\rho_f} \left(E_t \Delta g_{t+1} - \psi g_t \right) + \frac{(1+\psi_\tau)(1+\psi)}{1+\psi-\widetilde{\beta}\rho_\tau} E_t \Delta \tau_{t+1}
\end{aligned} \tag{89}$$

Therefore:

$$\begin{aligned}
q_t^n &= y_t^n - \frac{\varsigma}{(1+\psi)} E_t r_{t+1}^n + \underbrace{\frac{1}{1+\psi-\widetilde{\beta}\rho_f} \left(E_t \Delta g_{t+1} - \psi g_t \right)}_{-\frac{(1+\psi-\rho)}{1+\psi-\widetilde{\beta}\rho_f} f_t} + \underbrace{\frac{(1+\psi_\tau)(1+\psi)}{1+\psi-\widetilde{\beta}\rho_\tau} E_t \Delta \tau_{t+1}}_{-\frac{(1+\psi_\tau)(1+\psi)(1-\rho_\tau)}{1+\psi-\widetilde{\beta}\rho_\tau} \tau_t} \\
&= y_t^n - \frac{\varsigma}{(1+\psi)} E_t r_{t+1}^n - \frac{\left(1+\psi-\rho_f\right)}{1+\psi-\widetilde{\beta}\rho_f} f_t - \frac{(1+\psi_\tau)(1+\psi)(1-\rho_\tau)}{1+\psi-\widetilde{\beta}\rho_\tau} \tau_t
\end{aligned}$$

Substituting equation (89) into (87), we obtain the natural interest rate:

$$\begin{aligned}
\left(\widetilde{r}_t^n - \widetilde{\rho} \right) &= E_t \Delta y_{t+1}^n + \psi \left(q_t^n - y_t^n \right) - (1+\psi)(1+\psi_\tau) E_t \tau_{t+1} + \psi g_t - E_t \Delta g_{t+1} \\
\widetilde{r}_t^n &= \widetilde{\rho} + E_t \Delta y_{t+1}^n + \frac{\psi}{1+\psi-\widetilde{\beta}\rho_f} \left(E_t \Delta g_{t+1} - \psi g_t \right) + \\
&\quad + \frac{\psi(1+\psi_\tau)(1+\psi)}{1+\psi-\widetilde{\beta}\rho_\tau} E_t \Delta \tau_{t+1} - \frac{\psi\varsigma}{(1+\psi)} E_t r_{t+1}^n + \\
&\quad - (1+\psi)(1+\psi_\tau) E_t \tau_{t+1} + \psi g_t - E_t \Delta g_{t+1}
\end{aligned}$$

Recalling equation (86), we can manipulate the previous equation in order to obtain:

$$\widetilde{r}_t^n = \overline{\rho} + \widetilde{r}\widetilde{r}_t^n + \psi_f f_t + \psi_\tau \tau_t - \frac{\psi\varsigma}{(1+\psi)} E_t r_{t+1}^n$$

where $\overline{\rho} = \widetilde{\rho} - \rho = \log(1+\psi)$

$$
\begin{aligned}
\psi_f &= \frac{\psi \rho_f \left(1-\widetilde{\beta}\right)}{1+\psi-\widetilde{\beta}\rho_f} > 0 \\
\psi_\tau &= (1-\rho_\tau)\left[(1+\psi_\tau)(1+\psi) - 1 - \frac{\psi(1+\psi)(1+\psi_\tau)}{1+\psi-\widetilde{\beta}\rho_\tau}\right] \\
&= (1-\rho_\tau)\left[\frac{\psi_\tau(1+\psi) - \widetilde{\beta}\rho_\tau\left[(1+\psi_\tau)(1+\psi)-1\right]}{1+\psi-\widetilde{\beta}\rho_\tau}\right] > 0
\end{aligned}
$$

iff $\quad \psi_\tau > \frac{\psi}{1+\psi}\frac{\widetilde{\beta}\rho_\tau}{1-\widetilde{\beta}\rho_\tau}$.

1.8.4 Optimal monetary policy under discretion

Social Expansion Path The problem of an infinitively lived central banker is to choose a path for i_t, x_t and π_t to maximize:

$$
L = -\frac{1}{2}E_t\sum_{s=0}^{\infty}\beta^s\left(\pi_{t+s}^2 + \alpha_x x_{t+s}^2 + \alpha_s s_{t+s}^2\right)
$$

subject to equations(46), (45) and (47). Let be χ_t Υ_t and ϕ_t the Lagrangian multipliers associated with each constraint at time t, the first order conditions are:

$$
\begin{aligned}
\frac{\partial L}{\partial \pi_t} &= 0 \Longrightarrow -\pi_t + \Upsilon_t = 0 \\
\frac{\partial L}{\partial x_t} &= 0 \Longrightarrow -\alpha_x x_t + \chi_t - \kappa(1+\eta)\Upsilon_t - \phi_t = 0 \\
\frac{\partial L}{\partial i_t} &= 0 \Longrightarrow \frac{1}{1+\psi}\chi_t - \kappa\Upsilon_t = 0 \\
\frac{\partial L}{\partial s_t} &= 0 \Longrightarrow -\alpha_s s_t - \frac{\psi}{1+\psi}\chi_t + \phi_t = 0
\end{aligned}
$$

Rearranging the FOCs, we obtain the social expansion path:

$$
x_t = -\frac{\kappa\eta\pi_t + \alpha_s s_t}{\alpha_x} \quad \text{for} \quad t = 0, 1, 2... \tag{90}
$$

Note that if $\alpha_s = 0$, equation (47) is not taken into account by the Central Bank and this leads to a different SEP:

$$
\pi_t = -\frac{\alpha_x}{\kappa(\eta-\psi)}x_t \quad \text{for} \quad t = 0, 1, 2... \tag{91}
$$

Rational Expectation Equilibrium Solve jointly equations (46), (45), (47) and (56), we obtain the pseudo reduced form:

$$
\begin{aligned}
\pi_t &= \frac{\alpha_x\left(\widetilde{\beta}+\kappa\right)}{F}E_t\pi_{t+1}+\frac{\kappa\alpha_x\left(1-\delta\psi\right)}{F\left(1+\psi\right)}E_tx_{t+1}+\frac{\alpha_x\kappa\psi}{F\left(1+\psi\right)}E_ts_{t+1}-\frac{\alpha_x\xi\kappa\psi}{F\left(1+\psi\right)}E_ti_{t+1}+\\
&\quad+\frac{\kappa\alpha\left(1+\psi\right)}{F}g_t+\frac{\alpha_x}{F}u_t\\
x_t &= -\frac{\kappa\left(\widetilde{\beta}+\kappa\right)\left(\eta-\psi\right)}{F}E_t\pi_{t+1}-\frac{\kappa^2\left(\eta-\psi\right)\left(1-\delta\psi\right)}{F\left(1+\psi\right)}E_tx_{t+1}-\frac{\kappa^2\psi\left(\eta-\psi\right)}{F}E_ts_{t+1}+\\
&\quad+\frac{\psi\xi\kappa^2\left(\eta-\psi\right)}{F\left(1+\psi\right)}E_ti_{t+1}-\frac{\kappa^2\left(\eta-\psi\right)\left(\widetilde{\beta}+\kappa\right)}{F}g_t-\frac{\kappa\left(\eta-\psi\right)}{F}u_t
\end{aligned}
$$

$$
\begin{aligned}
s_t &= -\frac{\kappa\left(\widetilde{\beta}+\kappa\right)\left(\eta-\psi\right)}{F}E_t\pi_{t+1}-\frac{\alpha_x\left(1+\delta\right)+k^2\left(\eta-\psi\right)\left[1+\eta+\delta\left(\eta-\psi\right)\right]}{F\left(1+\psi\right)}E_tx_{t+1}+\\
&\quad+\frac{\alpha_x+\kappa^2\left(\eta-\psi\right)^2}{F\left(1+\psi\right)}E_ts_{t+1}-\frac{\xi\left[\alpha_x+\kappa^2\left(\eta-\psi\right)^2\right]}{F\left(1+\psi\right)}E_ti_{t+1}+\\
&\quad-\frac{\kappa^2\left(\eta-\psi\right)\left(1+\psi\right)}{F}g_t-\frac{\kappa\left(\eta-\psi\right)}{F}u_t
\end{aligned}
$$

$$
\begin{aligned}
i_t &= \left[1+\frac{\kappa\left(\widetilde{\beta}+\kappa\right)\left(\eta-\psi\right)}{F}\right]E_t\pi_{t+1}+\frac{\left[\alpha_x+\kappa^2\left(1+\eta\right)\left(\eta-\psi\right)\right]\left(1-\delta\psi\right)}{F\left(1+\psi\right)}E_tx_{t+1}+\\
&\quad+\left[\frac{\psi}{1+\psi}+\frac{\left[\alpha_x+\kappa^2\left(\eta-\psi\right)\right]\psi}{F}\right]E_ts_{t+1}-\frac{\xi\left[\alpha_x+\left(1+\eta\right)\kappa^2\left(\eta-\psi\right)\right]\psi}{F\left(1+\psi\right)}E_ti_{t+1}+\\
&\quad+\frac{\left(1+\psi\right)\left[\alpha_x+\left(1+\eta\right)\kappa^2\left(\eta-\psi\right)\right]}{F}g_t-\frac{\kappa\left(\eta-\psi\right)}{F}u_t
\end{aligned}
$$

where $F=\left[\alpha_x+\kappa^2\eta\left(\eta-\psi\right)\right]$.

1.8.5 Forward looking Interest rate

Deriving the parameters of the interest rate (7) respect to ψ we obtain:

$$
\begin{aligned}
\frac{\partial\Phi_\pi}{\partial\psi} &= -\frac{\alpha\kappa\left(\beta+\kappa\right)}{\left[\alpha_x+\kappa^2\eta\left(\eta-\psi\right)\right]^2}<0\\
\frac{\partial\Phi_s}{\partial\psi} &= \frac{\alpha^2+\eta\kappa^4\left(\eta-\psi\right)^2\left(1+\eta\right)+\alpha\kappa^2\left[\eta+2\eta\left(\eta-\psi\right)-\psi\left(2+\psi\right)\right]}{\left(1+\psi\right)^2\left[\alpha_x+\kappa^2\eta\left(\eta-\psi\right)\right]^2}>0
\end{aligned}
$$

if and only if

$$\psi = \frac{-\kappa^2 (1+\eta)\left(\alpha_x + \eta^2\kappa^2\right) + \sqrt[2]{\alpha_x \kappa^2 \left(\alpha_x + \kappa^2\eta^2\right)\left[\alpha_x + \kappa^2\eta^2 (1+\eta)^2\right]}}{\kappa^2 \left[\alpha_x - \eta (1+\eta)\kappa^2\right]} > 0$$

1.8.6 Proof of Proposition 1

Following the necessary and sufficient conditions of La Salle (1986) for a real $3x3$ matrix $N = M - I$ to be E-stable is that:

$$|\det N + trN| < 1 + m_1 \tag{92}$$

$$|m_1 - (\det N)(trN)| < 1 - (\det N)^2 \tag{93}$$

where $m_1 = \begin{vmatrix} m_{1,1} & m_{1,2} \\ m_{2,1} & m_{3,3} \end{vmatrix} + \begin{vmatrix} m_{1,1} & m_{1,3} \\ m_{3,1} & m_{3,3} \end{vmatrix} + \begin{vmatrix} m_{2,2} & m_{2,3} \\ m_{3,2} & m_{3,3} \end{vmatrix}$.
In our case we obtain:

$$\begin{aligned} \det N &= \frac{\psi \left[1 + \delta(\beta - 1) + \eta\right]}{1+\psi} \\ trN &= -1 + \beta - \delta + \eta + \frac{1+\delta}{1+\psi} \\ m_1 &= -\frac{1 + \delta(\beta-1) + \eta\psi(1-\delta)}{1+\psi} \end{aligned}$$

After some algebra, we observe that condition (92) implies:

$$\begin{aligned} \psi &< -\frac{\beta + 2\eta}{\beta(1+\delta) + 3\eta - \delta(4+\eta)} \\ \psi &> \frac{\beta}{\psi(1+\delta)(\beta+\eta)} \end{aligned}$$

Therefore for any structural parameters' values this condition does not hold.

1.8.7 Proof of Proposition 2

The necessary and sufficient condition for determinacy states that both eigenvalues of the matrix M must have eigenvalues with real negative part, i.e. they must lie inside the unit circle. The characteristic polynomial of our model takes the following form:

$$\begin{aligned} p(x) &= x^2 \left[x^2 + \frac{\kappa^2(\eta-\psi)\left[1 + \psi(1+\xi-\delta) + \eta(\xi\psi - 1)\right] - \alpha\left[1 + (\beta+\kappa)(1+\psi) - \psi\xi\right]}{\left[\alpha + \eta\kappa^2(\eta-\psi)\right](1+\psi)} x \right. \\ &\left. + \frac{\alpha\left[\beta + \kappa - \beta\xi\psi\right] - \kappa^2(\psi - \eta)}{\left[\alpha + \eta\kappa^2(\eta-\psi)\right](1+\psi)}\right] \end{aligned}$$

Since two eigenvalues are equal to zero, we study the sign of the remaining eigenvalues, using the necessary and sufficient condition of La Salle (1986).

In this case the characteristic polynomial can be rewritten as

$$p(x) = x^2 + a_1 x + a_0$$

where:

$$\begin{aligned} a_1 &= \frac{\kappa^2 (\eta - \psi) [1 - \eta + \psi(1 - \delta + \xi + \xi\eta)] - \alpha_x [1 + (\beta + \kappa)(1 + \psi) - \psi\xi]}{[\alpha_x + \eta\kappa^2 (\eta - \psi)](1 + \psi)} \\ a_0 &= \frac{\alpha_x [\kappa + \beta (1 - \xi\psi)] - \kappa^2 (\psi - \eta)}{[\alpha_x + \eta\kappa^2 (\eta - \psi)](1 + \psi)} \end{aligned}$$

Both eigenvalues lie inside the unit circle if and only if the Schur and Cohn's criterion is respected:

$$|a_0| < 1 \tag{94}$$
$$|a_1| < 1 + a_0 \tag{95}$$

Since $-\frac{\kappa^2(\eta-\psi)[1+\eta(1+\psi)]}{1+\beta+\kappa+(1+\beta\xi)\psi} < 0$, and if $-1+\beta+\kappa-(1+\beta\xi)\psi > 0$, condition (94) implies for $\alpha_x > 0$:

$$0 < \alpha_x < \frac{\kappa^2 (\eta - \psi) [1 + \eta (1 + \psi)]}{-1 + \beta + \kappa - (1 + \beta\xi)\psi} \tag{96}$$

Condition (95) gives rise to:

$$\frac{\kappa^2 (\eta - \psi) \{2 + (1 - \delta + \xi)\psi + \eta [\psi (\xi - 1) - 2]\}}{(2 + \psi)(1 + \kappa + \beta) - \xi (\beta + 1)} < \alpha_x < \frac{\kappa^2 (\eta - \psi) [(1 + \xi)(1 + \eta) - \delta]}{\kappa + (\beta - 1)(1 + \xi)}$$

In order to have $\delta = (2 + \eta - \Phi)(1 - \beta)$, $\xi = \frac{(1-\beta)}{(\Phi-1)} > 0$, we impose $1 < \Phi < 2 + \eta$ [18], the left side inequality is always negative for any reasonable $0 < \kappa < 1$, $\eta > 1$ and $0 < \beta < 1$. Therefore, the second condition requires that for any $\alpha_x > 0$:

$$0 < \alpha_x < \frac{\kappa^2 (\eta - \psi) [(1 + \xi)(1 + \eta) - \delta]}{\kappa + (\beta - 1)(1 + \xi)} \tag{97}$$

Therefore, the necessary and sufficient conditions are:

$$0 < \alpha_x < \min \left\{ \frac{\kappa^2 (\eta - \psi) [1 + \eta (1 + \psi)]}{-1 + \beta + \kappa - (1 + \beta\xi)\psi}, \frac{\kappa^2 (\eta - \psi) [(1 + \xi)(1 + \eta) - \delta]}{\kappa + (\beta - 1)(1 + \xi)} \right\}$$

[18] We derive this condition from the definitions for δ and ξ obtained in the previous section.

1.8.8 When asset price targeting matters

Reduced form

$$
\begin{aligned}
x_t &= \frac{\left(\widetilde{\beta}+\kappa\right)}{\Theta}E_t\pi_{t+1}+\frac{\alpha_s\left(1+\delta\right)+\kappa^2\eta\left(\delta\psi-1\right)}{\Theta\left(1+\psi\right)}E_tx_{t+1}-\frac{\left[\alpha_s+\eta\kappa\psi\right]}{\Theta\left(1+\psi\right)}E_ts_{t+1}+\\
&\quad+\frac{\xi\left[\alpha_s+\psi\eta\kappa^2\right]}{\Theta\left(1+\psi\right)}E_ti_{t+1}-\frac{\eta\kappa^2\left(1+\psi\right)}{\Theta}g_t-\frac{\eta\kappa}{\Theta}u_t\\
\pi_t &= \frac{\left(\alpha_x+\alpha_s\right)\left(\widetilde{\beta}+\kappa\right)}{\Theta}E_t\pi_{t+1}+\frac{\kappa\left[\left(\alpha_x+\alpha_s\right)\left(1-\delta\psi\right)-\alpha_s\eta\psi\left(1+\delta\right)\right]}{\Theta\left(1+\psi\right)}E_tx_{t+1}+\\
&\quad+\frac{\kappa\left[\left(\alpha_x+\alpha_s\right)\psi-\eta\alpha_s\right]}{\Theta\left(1+\psi\right)}E_ts_{t+1}+\frac{\kappa\xi\left[\eta\alpha_s-\left(\alpha_x+\alpha_s\right)\psi\right]}{\Theta\left(1+\psi\right)}E_ti_{t+1}+\\
&\quad+\frac{\left(\alpha_x+\alpha_s\right)\left(1+\psi\right)}{\Theta}g_t+\frac{\left(\alpha_x+\alpha_s\right)}{\Theta}u_t
\end{aligned}
$$

$$
\begin{aligned}
s_t &= \frac{\left(\widetilde{\beta}+\kappa\right)}{\Theta}E_t\pi_{t+1}-\frac{\alpha_x\left(1+\delta\right)+\eta\kappa^2\left[1+\eta+\delta\left(\eta-\psi\right)\right]}{\Theta\left(1+\psi\right)}E_tx_{t+1}+\\
&\quad+\frac{\alpha_x+\kappa^2\eta\left(\eta-\psi\right)}{\Theta\left(1+\psi\right)}E_ts_{t+1}+\frac{\xi\left[\alpha_x+\kappa^2\eta\left(\psi-\eta\right)\right]}{\Theta\left(1+\psi\right)}E_ti_{t+1}+\frac{\eta\kappa^2\left(1+\psi\right)}{\Theta}g_t+\\
&\quad+\frac{\eta\kappa\left(\psi-1\right)}{\Theta\left(1+\psi\right)}u_t\\
i_t &= \left[1+\frac{\eta\kappa\left(\widetilde{\beta}+\kappa\right)}{\Theta}\right]E_t\pi_{t+1}+\frac{\left[\alpha_x+\eta\kappa^2\left(1+\eta\right)\right]\left(1-\delta\psi\right)-\delta\alpha_s}{\Theta\left(1+\psi\right)}E_tx_{t+1}+\\
&\quad+\left[\frac{\psi}{\left(1+\psi\right)}+\frac{\alpha_s+\psi\eta\kappa^2}{\Theta\left(1+\psi\right)}\right]E_ts_{t+1}-\left[\frac{\xi\psi}{\left(1+\psi\right)}+\frac{\xi\left(\alpha_s+\eta\kappa^2\psi\right)}{\Theta\left(1+\psi\right)}\right]E_ti_{t+1}\\
&\quad+\frac{\alpha_x+\alpha_s+\eta\kappa^2\left(1+\eta\right)\left(1+\psi\right)}{\Theta}g_t+\frac{\eta\kappa}{\Theta}u_t
\end{aligned}
$$

where $\Theta=\left(\alpha_x+\alpha_s+\kappa^2\eta^2\right)$.

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