Managing Migration through Quotas: an Option-theory Perspective

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Abstract
Recent European Legislation on immigration has revealed a particular paradox on migration policies. On one hand, the trend of recent legislation points to the increasing closure of frontiers (OECD 1999, 2001, 2004), also by using immigration quotas. On the other hand, there is an increase of regularization, i.e., European policies are becoming less tight. Our aim here is to study these counterbalanced and opposite policies in European immigration legislation in an unified framework. To do this, we have used a real option approach to migration choice that assumes that the decision to migrate can be described as an irreversible investment decision where quotas represent an upper bound limit. Our results show the paradox of counterbalancing immigration policies is not odd but it could be in line with an optimal policy to control migration inflow. In particular, we show that if the government controls the information related to the immigration quota system it could delay the mass entry of immigrants maintaining, in the long run, the required immigration stock and controlling the flows in the short-run.

JEL Classification Numbers: F22, J15, J61, O15, R23.

Keywords: Immigration, Real Option, Quota System.

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1 Introduction

While barriers to international trade and capital mobility have been largely removed, labour markets are still the most tightly regulated areas of economic activity (Faini et al., 1999). In this respect, Boeri and Brücker (2005), studying European migration, showed that rules for legal immigration into the EU from third countries are getting tighter and tighter: "since 1990 there have been 92 reforms of national migration policies in to the EU-15, that is, more than five reforms per year. Most of these reforms are marginal in that they adjust specific provisions rather than revising the overall regulatory framework. Furthermore, seven reforms out of ten tighten regulations, for example, by increasing procedural obstacles faced by those applying for visas, reducing the duration of work permits or making family reunification more difficult", or by introducing an immigration quota system\(^1\). In particular, this latter immigration policy has been adopted by certain European countries (Austria, France, Greece, Italy, Portugal, Spain, UK) to control migration inflow better and was suggested at the meeting of the EU Justice and Home Affairs ministers in Stratford-upon-Avon in late October 2006. Nevertheless, despite this evidence, another aspect related to migration policy has revealed a peculiar paradox of migration policies. Since 1990, there have been 26 (39 since 1973) one-shot regularization programs in 10 EU countries (Jachimowicz et al., 2004; Sunderhaus, 2007)\(^3\). Therefore, on one hand as a result of increased labour market competition and concerns about terrorism the trend of recent legislation over immigration points to increased closure of frontiers (OECD 1999, 2001). On the other hand, there are more regularization programs which, as anticipated by the immigrants, relax the effect of immigration quotas and make the European policies less tightened. Therefore, what kind of policy is best to control immigration? As many countries have adopted simultaneously two kinds of opposite immigration policies, at a first glance, it seems that the legislator has no clear idea of what kind of policy is best to control immigration. Our aim in this paper is to answer this question, by investigating the counterbalancing immigration policies in European immigration legislation in an unified framework.

By using a recent approach to migration choice that assumes that the decision to migrate can be described as an investment decision (Sjaastad, 1962), we have approached the above question by extending recent results obtained by Bartolini\(^4\) (1993; 1995). Bartolini shows that a competitive market reacts to limit\(^5\) aggregate investment by generating recurrent runs as the total investment approaches its limit. That is, the existence of quotas on aggregate investment may induce endogenous and recurrent asset runs so that the quotas are immediately filled. In the specific, the aggregate investment evolves smoothly over time, driven by market conditions, until it reaches an upper threshold where it shows a jump that fills the quotas.

We shows that introducing some uncertainty over the quota in a competitive migration market, the entry run tends to vanish. As each agent is not able to perfectly foresee the real quota, he acts as the quota did not exist. The entry process tends to be smooth and has no jumps. The ambiguity over the true quota reduces the entry runs by potential immigrants, allowing the government to obtain, in the long run, the required immigration stock and to control flow in the short-run. In this context, the presence of regularization programs that make the agents unable to perfectly foresee the real quota is not any more a paradox, but it could be useful for the planner to control immigration inflow.

This paper is related to past research that applies the real option approach to migration phenomena. In this respect, Burda (1995), showed that individuals prefer to wait before migrating, even if the present value of the wage differen-
tial is positive, because of the uncertainty and the sunk costs associated with migration. Subsequently Khwaja (2002) and Anam et al., (2007) developed Burda’s approach by describing the role of uncertainty in the migration decision. Another work that uses real option in migration is Feist (1998), in which the author analyses the option value of the low-skilled workers to escape to the unofficial sector if welfare benefits come too close to the net wage in the official sector. Three recent papers (Moretto and Vergalli, 2008; Vergalli, 2007; Vergalli, 2008) applied the real option framework to the analysis of migration dynamics, focussing on the role of communities and network to explain mass migration.

In the first part, we describe what happens in migration dynamics if the authority imposes a determined and known quota on the immigration entries. In the second part, we show that the introduction of noise over the quota system delays mass entry. This uncertainty can be created either by announcing policies followed by different action by the government or by introducing different policies relaxing or tightening the conditions to immigrate. In both the cases this uncertainty could also depend on governments with unstable majorities, that probably is expressed in counterbalancing migration policies. This fact could also explain why recent legislation on immigration moves towards the two counterbalancing directions explained above: this ambiguity related to quotas and regularization programs could increase especially in countries with unstable majorities. The result is that in this case, the migration inflow becomes smooth independent of the particular policy adopted. Indeed, as stressed in the OECD International Migration Outlook (2006), "In practice, however, the national limits and associated quotas have been less than the numbers requested by employers and have proven to be significantly under actual labour market needs, if the extent of regularisations of persons with employment contracts is any indication [...] the regular lack of concordance between the programmed migration levels and labour market needs meant that in practice, the levels had become almost irrelevant. Employers may well have become accustomed to a situation in which they could hire outside legal channels with relative impunity, with a reasonable probability that the hiring would be formally recognised a few years hence through regularisation".

This paper is organised as follows. Section 2 summarises the evolution of national immigration policies. Section 3 presents the model and the basic assumptions. Section 4 develops the theoretical framework with known quota. Section 5 develops the theoretical framework with unknown quota and the main results. Section 6 summarises the conclusions. Finally, the Appendix contains the proofs omitted in the text.

2 Evolution in National Immigration Policies

Immigration policies could be tightened by using different criteria. In this respect, Boeri and Brucker (2005) developed an aggregate policy index that describes "the trend in migration policies". The index is obtained by taking the average of the following seven indicators: 1) admission requirements; 2) number of administrations involved; 3) length of first stay; 4) quotas; 5) residence requirement; 6) years to obtain a permanent permit; 7) asylum policy. According to their analysis the national immigration policies are becoming more tighten. There is no doubt that all these seven policies are the mirror of European immigration policy. Nevertheless, for some of these policies we should distinguish between short and long term effects also between their effects on migration flow and/or migration stocks. For example, let us consider the effect of a quota in...
line with government legislation: on one hand the quotas may be able to control migration stocks in the long run, on the other hand they can trigger off some run-entry mechanisms that could defeat any control of inflow and its speed. That is, quotas are useful to control, at least in the long run, the total number of immigrants (stock) but not the entry speed (flow). Moreover, this effect is stronger when the quota is perceived by the immigrants as the last chance to enter: they all hurry to enter the host country. In table 1 we show the European countries that have recently introduced this kind of policy, being Austria, Czech Republic, France, Greece, Italy, Portugal, Slovenia, Spain, Switzerland and the United Kingdom.

<table>
<thead>
<tr>
<th>Country</th>
<th>Quota</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>In 1990, a quota for the employment of foreigners was introduced, defined as a maximum share of foreign workers in the total workforce. The Residence Act 1991 has the objective to control immigration. It defines the quantitative and qualitative criteria for the potential residence of different groups of foreigners: definition of quota for certain sub-groups of immigrants (family members); The Aliens Law 1997 (Fremdenzugsrechtsordnung) came into effect in 1999. It regulates the conditions for entry and residence in the country. The key concept of the reform is: &quot;Integration Before New Immigration&quot;. Reduction of immigration quotas; The Aliens Law 2002 changes the conditions for entry and residence in the country; it provides for a stricter system of immigration control and it tightens the quota system. Key professional are not subject to quotas, don't have to fulfill the integration agreement and can obtain a residence permit with the authorization to work.</td>
<td></td>
</tr>
</tbody>
</table>
| Czech Republic| The Czech government launched in 2002 a pilot program for the active selection of the qualified foreign workers. The quota were established for the first two years: 400 and 1200 persons for 2003 and 2004 respectively. The Employment and Work of Aliens Act is the main act regulating the immigration in the Czech Republic. Government.
| France        | In 2000, the government has been required to submit to parliament an annual report specifying the number and kind of residence permits to be authorized over a three-year period. The draft bill avoids using the word "quotas", but articles say the provision amounts to a "quota system". In 2007 government decided to adopt decreases on immigration quotas: by profession, category and, annually, by regions of the world. |
| Greece        | The Law 1857/1991 defines for the first time the legal situation of migratory and refugees. It's an attempt to modernize the relevant legislation on issues of entrance, exit, stay, settlement, employment and expulsion of aliens. The Law 2810/2001 regulates the procedure concerning work permits: the Ministry of Employment and Social Solidarity must define the criteria for the issuance of work permits. This is the first law that bruits the idea of quotas. With ministry of Employment's decree 15 February 2006, were defined the criteria. This is the first law that bruits the idea of quotas. The Employment and Work of Aliens Act is the main act regulating the immigration in Greece. Government. |
| Italy         | The Law 39/1990 (Matelli Law) regulates the entry and residence of non-EU citizens: migration begins to be considered as a social phenomenon. The law defines the conditions to grant entry permits for working reasons: the Government has to draw up a yearly plan instead of referring to pre-defined criteria. This is the first law that bruits the idea of quotas. With Ministry of Employment's decree 15 February 2006, were defined the non-EU immigrant quotas. |
| Portugal      | The Decree-Law 34/2001 introduces a system of quotas to regulate the entry of non-EU citizens: migration begins to be considered as a social phenomenon. The law defines the conditions to grant entry permits for working reasons: the Government has to draw up a yearly plan instead of referring to pre-defined criteria. This is the first law that bruits the idea of quotas. With Ministry of Employment's decree 15 February 2006, were defined the non-EU immigrant quotas. |
| Slovenia      | The Employment and Work of Aliens Act is the main act regulating the economic migration in Slovenia. It sets the policy priorities as well as the maximum number of foreign workers. The Residence Act 1993 has the objective to control immigration. It defines the quantitative and qualitative criteria for the potential residence of different groups of foreigners: definition of quota for certain sub-groups of immigrants (family members); |
| Spain         | The quota system is the basic mechanism used to manage the labour immigration in Spain. It was used in the years 1990-1995, 1997-1999 and since 2002. The aim of the quotas was to direct the immigrants to the labour market sectors which suffered from shortages. |
| Switzerland   | The Swiss government relaxed its immigration laws on 01 June 2007. Quotas will remain in place for these countries until at least 2011. The previous system allowed 15,000 permanent residence permits to be granted annually for people who had a job contract for more than one year. This quota was quickly cancelled each year. Short-term permits, allocated at 115,000 per year, were less popular with only 55%-60% being used. |
| United Kingdom| Working Holiday Makers – around 46,000 young people from Commonwealth countries (17-27 years old) are allowed to come to Britain and take up non-professional job for up to 2 years; Seasonal agricultural workers – for students, mainly from Central and Eastern Europe, who arrive within a set quota (Fresh Manufacturing Fish and Meat sector in the context of Sector Based Scheme Permit); As pairs – around 15,000 per annum; Domestic workers – around 15,000 per annum (Spencer 2002). |

Table 1: immigration quota system adopted in some countries

To be complete in the analysis of migration policies, we must also add another instrument that governments can use to control migration: regularization programs which by definition, relax the effect of immigration quotas and modify at the same time immigrant flow and stocks. So, looking at European legislation, it is possible to see that several countries impose both admission requirements
and quotas to reduce entry. Nevertheless, they also adopt frequent regularization programs. In Table 2 we can see the programs adopted in Europe since 1973\(^9\). Since 1990, there have been 26 (39 since 1973) one-shot regularization programs in 10 EU countries.

<table>
<thead>
<tr>
<th>Country</th>
<th>Regularizations</th>
<th>Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>2</td>
<td>1996, 1999</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>1</td>
<td>2001</td>
</tr>
<tr>
<td>Total</td>
<td>39</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Regularization Programs in 10 EU countries.

3 The Model

3.1 The basic assumptions

For simplicity, the model uses the familiar terminology of an agents' entry decisions under uncertainty\(^10\). Consider the immigration decision of individuals in a host country subject to an uncertain wage gap. Let us summarize the main assumptions:

1. At any time \(t\), a potential immigrant may decide to migrate ("entry"). Individuals are risk-neutral and discount the future income at the constant discount rate \(\rho\).

2. Each individual can migrate by committing irrevocably to a flow cost \(w\) or undertaking a single irreversible investment which requires an initial sunk cost \(K = w/\rho\).

3. \(n_t\) is the number of individuals that are in the host country at time \(t\), each yields a flow of income\(^11\):

\[
\pi(\theta_t, n_t) = u(n_t) \theta_t
\]

where \(\theta\) is a multiplicative labour market-specific shock. We can consider, in a simpler setting, \(u(n_t)\) as the inverse demand function (Dixit and Pindyck, 1994, ch. 9; Bartolini, 1993; Nielsen, 2002) or as a reduced form of a more general benefit function (Dixit and Pindyck, 1994, ch. 11; Dixit, 1995; Grenadier, 2002; Moretto and Vergalli 2008). Time is continuous, \(t \in [0, \infty)\), and suppressed if not necessary.

4. The function \(u(n)\) is continuously differentiable in \(n\) with the usual properties.

\[
u(n) > 0, \quad u'(n) < 0
\]

\[
\lim_{n \to 0} u(n) = +\infty \quad \text{and} \quad \lim_{n \to N} u(n) = u > 0
\]
where $\bar{N} \leq \infty$ can be interpreted as the upper saturation level of the ethnic community in the host country. So, a positive reserve "utility" $u$ means that for each immigrant the benefits from migration (even in the worst case) are higher than the costs (in the wider sense) to migrate.

5. All individuals are identical and their size $dn_t$ is infinitesimally small with respect to the labour market in the host country.

6. The labour market-specific shock follows a geometric diffusion process:

$$d\theta_t = \alpha \theta_t dt + \sigma \theta_t dW_t \quad \text{with} \quad \theta_0 = \theta \quad \text{and} \quad \alpha, \sigma > 0$$

(2)

where $\alpha < \rho$ and $dW_t$ is the increment to a Wiener process, satisfying $E(dW_t) = 0$ and $Var(dW_t) = dt$.

In the next section, we assume that the quota is known to the immigrants. The existence of a limit on the aggregate level of migration induces an externality among the benefit functions of different immigrants, which causes a possible divergence between the socially-optimal and profit-maximizing policies. Then, in section 4 we will relax this assumption, by assuming that the immigrants might unknown the true quota.

3.2 Solution with defined quota

For the first result we have added the following assumption:

7. There exists an exogenous determined quota $N < \bar{N}$ on $n$, which is announced by the government and is known to all the potential immigrants.

To determine the migrant’s optimal entry policy, the first thing to do is to find his/her value given each individual’s optimal future entry policy. Let us consider the value of an immigrant $V(\theta, n, N)$, that is active in the market, as the expected discounted stream of income:

$$V(\theta, n, N) = \max_\tau E_0 \left[ \int_0^\infty e^{-\rho t} \pi(\theta_t, n_t) dt - J_{[\tau=\tau]} K \mid n_0 = n, \theta_0 = \theta \right]$$

(3)

where $J_{[\tau=\tau]}$ is the indicator function and the expectation is taken considering that the number of active immigrants may change over time by new entry. The solution to (3) can be obtained starting within a time interval within which no new entry occurs. Over this interval the number of immigrants $n$ is fixed and $V(\theta, n, N)$, must satisfy the no-arbitrage requirement where time is suppressed if not necessary:

$$\pi(\theta, n) + E[\frac{dV(\theta, n, N)}{dt}] = \rho V(\theta, n, N)$$

(4)

Assuming $V(\theta, n, N)$ to be a twice-differentiable function with respect to $\theta$ and using Itô’s Lemma to expand $dV(\theta, n, N)$, the solution of (3) is given by the following differential equation (Dixit and Pindyck, 1994, p. 179-180):

$$\frac{1}{2} \sigma^2 V_{\theta\theta}(\theta, n, N) + \alpha V_\theta(\theta, n, N) - \rho V(\theta, n, N) + \pi(\theta, n) = 0$$

(5)

The general solution of (5):

$$V(\theta, n, N) = B(n, N) \theta^\beta + \frac{\theta u(n)}{\rho - \alpha}$$

(6)
Where the last term \( \left( \frac{\theta u(n)}{\rho - \alpha} \right) \) represents the value of migration in the absence of new entry\(^{18}\), then \( B(n, N) \theta^2 \) is the correction of the migration’s value due to the new entry and \( B(n, N) \) must therefore be negative. Obviously, a last boundary condition applies to the value of the \( N^{th} \) entry. The value of the \( N^{th} \) entry should converge to the value of a migration calculated by keeping the number of immigrants fixed at \( N \), i.e. \( V(\theta, n, N) = \frac{\theta u(N)}{\rho - \alpha} \). This implies that:

\[
B(N, N) = 0.
\] (7)

If the benefit value function (6) is known, the optimal migration policy implies that the return from migration must be at least equal to cost \( K \) at the entry point. In other words, we need to find the trigger value \( \theta^*(n) \) (i.e. the value of the labour demand shock) at which the \( n^{th} \) migrant is indifferent between immediate entry or waiting another instant. This trigger should be calculated bearing in mind that \( N \) is the upper limit oltre il quale nessuna altra entrata è consentita.\(^{19}\) This is defined in the following proposition:

**Proposition 1** The benefit-maximizing entry policy in a market with a quota \( N \) is given by:

\[
\theta^*(n) = \frac{\beta}{\beta - 1} (\rho - \alpha) \frac{K}{u(n)} \text{ for } n = (0, n^*]
\]

\[
\theta^*(n^*) = \theta^*(N) \text{ for } n = [n^*, N]
\]

where \( \theta^*(N) = (\rho - \alpha) \frac{K}{u(N)} \).

**Proof.** See Bartolini (1993) and the Appendix.

By Proposition 1, the entry policy is efficient until a number \( n^* < N \) of individuals have entered the market. At that point a migration run takes place and the residual quota is instantly filled. As proved by Bartolini (1993), \( n^* \) is determined by the fact that it splits the interval \((0, N]\) into two subintervals. In the first interval, the individuals enter by following the usual matching value and smooth pasting conditions, i.e. \( V(\theta^*(n), n, N) = K \) and \( V_\theta(\theta^*(n), n, N) = 0 \), so that \( \frac{d\theta^*(n)}{dn} > 0 \) (see the Appendix); in the second interval, the individuals migrate by a "run" until the whole quota is instantly filled, i.e. \( \frac{d\theta^*(n)}{dn} = 0 \), while, from (8) and (9), \( n^* \) is given by:

\[
\frac{u(n^*)}{u(N)} = \frac{\beta}{\beta - 1}
\] (10)

The insights from Proposition 1 are shown in Figure 1, below. In particular:

i) In the first quadrant on the left, on the abscissa, there is the entry value for different \( \theta \) and \( n \) levels. The migration value of the first \( n^* \) immigrants follows the S-shaped curve typical of model of investment hysteresis (Dixit and Pindyck, 1994, p. 220). These curves are tangential to the barrier (i.e., the entry cost) \( K \) and describe the value of migration as long as it fluctuates under the \( K \) level. The last \((N - n^*) \) curves cross the level \( K \), and all of them must cross \( K \) at the same level of fundamentals \( \theta \).\(^{20}\) Whenever \( V(\theta, n, N) \) reaches \( K \), the number of immigrants increases, shifting the current curve rightward. When \( n \) reaches \( n^* \), a large change in \( n \) shifts the current curve from \( V(\theta, n^*, N) \) to \( V(\theta, N, N) \).

ii) The second quadrant on the right shows the threshold levels for different numbers of immigrants. Below or to the right of the curve no migration
occurs because at a given level \( \theta(n) < \theta^*(n) \), the benefit for each potential immigrant is lower than the cost faced to migrate. This means that above the curve it is optimal to migrate. A wave of migrants will enter in a lump to move the benefit level immediately to the threshold curve. In the region below the curve the optimal policy is inaction. But the shock can cross the trigger for different numbers of individuals, \( n \). To appreciate the explanation of Figure 1, let us consider a sequential entry starting at \( n < n^* \). If the initial size of the community is \( n < n^* \), we can expect migration to work in the following way. For any fixed \( n \), if the benefits climb to a certain level \( \pi^* = u(n)\theta^*(n) \), migration becomes feasible, the network size increases from \( n \) to \( n + dn \) and the benefits go downward along the function \( u(n) \). If the size of the community is \( n^* \leq n \leq N \), when the shock hits the threshold \( \theta^*(n) \), then the quota is instantaneously filled and a mass \( (N - n) \) of individuals enter and the benefits climb to \( \pi^* = u(N)\theta^*(N) \). Therefore, until \( n^* \) the individuals migrate in a smooth manner, but between \( n^* \) and \( N \) they enter in a mass because for \( (N - n^*) \), individuals the threshold level is the same.

**Figure 1**: Optimal threshold levels with known quota \( N \)

Summarizing, with free entry, labour market competition generates a run that fills the quota when a fraction \( n^*/N \) has been filled. Until then, the entry policy is identical to the case without a quota. Immigrants initially enter at the optimal pace, knowing that all the potential benefits will dissipate by the early entry of the last \( (N - n^*) \) individuals.

### 4 Solution with undetermined quota

So far we have analysed the optimal policy with a fixed-known quota on the number of individuals allowed to enter in the host country, but what happens if the limit were perceived to be uncertain by immigrants? To introduce uncertainty over the quota, we replace assumption (7) with the following assumption:

7 **bis.** Each individual does not know the exact limit over the stock imposed by the government. However he/she knows that the quota is continuously distributed and drawn from a common distribution function
\[ F(N) = \Pr(\mathcal{N} < N) \] which is strictly increasing on the interval \([0, \overline{N}]\), where \(\overline{N}\) is the upper support of the distribution of \(\mathcal{N}\), and it has a continuous differentiable density \(f(N)\).

Further, we assume that each individual makes rational conjectures about the distribution of \(\mathcal{N}\) over time. More specifically, as new individuals decide to migrate, the individual will update his/her conjecture about \(\Pr(\mathcal{N} < N)\). As time goes by and \(n\) increases the potential immigrant learns that the probability of hitting the quota is higher. The individual then observes the realization of the state variable \(n\) and updates its conjecture by using \(G(N; n) = \frac{F(N) - F(n)}{1 - F(n)}\) which is strictly increasing on the interval \([n, \infty)\), with density \(g(N; n) = f(N)\).

Since now the individuals do not know the true quota, the value of their decision cannot be defined by (6). In particular, the last boundary condition (7) calculated by keeping the number of immigrants fixed at \(N\), should be substituted by:

\[ \lim_{n \to \overline{N}} E(B(n)) = 0 \] (11)

where the expectation operator is taken with respect to the random variable \(\mathcal{N}\).

As before, also in this case, we should find the threshold level \(\theta^*(n)\) that corresponds to the optimal entry process. Taking into account condition (11), we can prove the following proposition:

**Proposition 2** The benefit-maximizing entry policy in a market with an unknown quota is given by:

\[
\begin{align*}
\theta^*(n) &= \frac{\beta}{\beta - 1} (\rho - \alpha) \frac{K}{u(n)} \quad \text{for } n = (0, n^{**}] \\
\theta^*(n^{**}) &= \frac{\beta}{\beta - 1} (\rho - \alpha) \frac{K}{u(n^{**})} \quad \text{for all } n > n^{**}
\end{align*}
\] (12)  (13)

where \(n^* < n^{**}\).

**Proof.** see the Appendix.

Proposition 2 states that the entry policy is efficient until a number \(n^{**}\) of immigrants has entered the market. At that point, a migration run starts until the true (unknown) quota is reached. That is, since the true quota is unknown, the migration run continues until the government stops entries since the predefined limit has been reached. In addition, by looking at (12) and (13) it is evident that the optimal trigger \(n^{**}\) is obtained by considering all support of the distribution \(F(N)\), i.e., each individual acts as if the quota did not exist and the utility to remain out of the country were close to zero:

\[ \frac{u(n^{**})}{u} = \frac{\beta}{\beta - 1} \] (14)

Finally, by direct inspection of (10) and (14), it is immediate obvious that \(n^{**} < n^*\) as long as \(u(N) > u\).

To interpret these results, let us look at Figure 2. In particular, in the quadrant on the left we have the value of the immigrants on the horizontal axis and the threshold level on the vertical axis. In the quadrant on the right, we have the threshold level on the vertical axis and the number of immigrants in the host country on the horizontal axis. The red line represents the optimal trigger as a function of the number of immigrants. By comparing Figure 1 and
Figure 2, we can see that while, without uncertainty, the optimal threshold level flattens at \( n^* \), under uncertainty the competitive run starts at \( n^{**} > n^* \) and continues until the true (unknown) limit is reached.

If different political parties alternate in the government of a country, they probably express different policies. In particular they probably have different and counterbalancing migration policies. This ambiguity related to quotas and regularization programs could increase especially in countries with unstable majorities. The result is that in this case, the migration inflow becomes smooth independent of the particular policy adopted.

A policy remark about this result is in order. Comparing the two rules (10ne) and (14) we get:

\[
\frac{u(n^{**})}{u(n^*)} = \frac{u}{u(N)}
\]

which means that the ratio between \( n^{**} \) and \( n^* \), does not depend on the distribution of the quota \( F(N) \) but only on the ratio between \( u \) and \( u(N) \). If adopting repeated regularization programs a country is able to generate noise over the true quota \( N \), and to install the idea in the immigrants that the labour market’s saturation level might increase, the entry jump is moved forward. That is, the entry run happens at a higher size \( n^{**} \), corresponding to a higher benefit level \( u(n^{**})\theta^*(n^{**}) \), which means that the quota is fullfilled later. In other words, if the government’s aim is to delay migration waves and smoothing entries, it can do it by controlling informations regarding the immigration quota.

5 The effect of labour market uncertainty

Our model allows for a deeper study of the effect of uncertainty over labour demand on entry policy as well as on the optimal triggers \( n^* \) and \( n^{**} \). From (8) (or 12), (10) and (14) we can show that: \(^{24}\)

\[
\frac{d\theta^*(n)}{ds} > 0
\]
and
\[ \frac{dn^*}{d\sigma} < 0 \quad \text{and} \quad \frac{dn^{**}}{d\sigma} < 0 \]  
(17)

As anticipated by the Real Option Theory, an increase in the labour demand volatility ($\sigma$) increases the $\frac{\beta}{\sigma}$ ratio which, in turn, rises the threshold of $\theta^*$ ($n$) for any given number of immigrants $n$. In this sense, greater uncertainty implies less willingness to migrate. However, as shown by (17), greater uncertainty magnifies the competitive effect, reducing the size that triggers the entry run. Therefore, depending on what kind of effects prevails, we may get two entry patterns as shown in Figure 3. If the uncertainty effect is soft, then the competition effect, coming from a decrease of the crucial level $n^{**} < n^{*}$ is stronger than the entry delay caused by the raise of the threshold level (lower bold dotted line in figure). Entry is pushed forward because of the decrease of competition and, although we experiment a reduction of immigration flow, the average time to reach the government’s predefined limit can be substantially reduced. On the contrary, if the effect of uncertainty is strong, the time delay of migration entry (higher bold dotted line in figure) is stronger than the reduction of competition: in this case there is a reduction of migration inflow and an increase of the average time to reach the government’s predefined quota.

![Figure 3. Undetermined quota: threshold levels for increasing variance](image)

6 Conclusion

Recent European legislation on immigration reveals a peculiar paradox on migration policies. On one hand, as a result of increased labour market competition and concerns about terrorism, the trend of recent legislation over immigration points to increasing frontier closure (OECD 1999, 2001). From the other, there is an increase of regularization, that is the European policies become less tightened. We have tackled these counterbalancing and opposite policies by using a real option approach for migration choice that assumes that the decision to migrate can be described as an irreversible investment decision (Burda, 1995; Moretto and Vergalli, 2008). The first result agrees with economic literature...
(Bartolini, 1993) and shows that if a government imposes a quota over the migration stock, the potential immigrant rushes towards the host country because they are afraid of being left out. If, however, a government is ambiguous about the quota system, that is, alternating a tightening of admission requirements and quotas to regularization programs, it can delay the mass entry of immigrants. If this is the case then the counterbalancing immigration policies used by European countries is not so odd. It could be useful to indirectly delay immigration waves. Moreover, if certain governments have unstable majorities, that probably is expressed in counterbalancing migration policies, the migration inflow could become smooth independent of the particular policy adopted, but, also as a consequence of this political ambiguity. Furthermore, if a government’s aim is to delay entry migration waves, it could control it by causing noise on information relating to immigration quotas. In conclusion, between the two policies adopted (tightening or reducing the rules for legal immigration) there exists a third policy that is to alternate tightening and reduction in order to create uncertainty over the quota system that may help to control entry better.
A Proof of proposition 1

A family of solutions of (5) is given by:

\[ V(\theta, n, N) = A(n, N) \theta^\gamma + B(n, N) \theta^\beta + \hat{V}(\theta, n) \]  
(18)

where \( \beta \) and \( \gamma \) are the positive and negative roots of the quadratic equation in \( \lambda : \) 
\[ \frac{\sigma^2}{2} \lambda (\lambda - 1) + \alpha \lambda - \rho = 0 \] 
with \( 1 < \beta < \frac{\rho}{\alpha} \) and \( A(n, N) \) and \( B(n, N) \) are the two families of integration constants; \( \hat{V}(\theta, n) \) is chosen as the discounted expectation of flow payoff calculated by keeping the number of immigrants fixed at \( n \):

\[ \hat{V}(\theta, n) = E_0 \left[ \int_0^\infty \pi(n, \theta) e^{-\rho t} dt \mid \theta_0 = \theta \right] = \frac{\theta u(n)}{\rho - \alpha} \]  
(19)

Because the probability of entry tends to zero as \( \theta \) tends to zero, one boundary condition is that \( \lim_{\theta \to 0} V(\theta, n, N) = 0 \), this implies that \( A(n, N) = 0 \), and then the equation:

\[ V(\theta, n, N) = B(n, N) \theta^\beta + \frac{\theta u(n)}{\rho - \alpha} \]  
(20)

in the text. The coefficient \( B(n, N) \) can be determined by using the following suitable set of boundary conditions:

1. First, by competitive pressure, the value-matching condition requires the value of being entered is equal to the entry cost \( K \) at \( \theta = \theta^*(n) \), i.e., in equilibrium immigrant expect zero profit at entry (Dixit and Pindyck, 1994, ch.8).

\[ V(\theta^*(n), n, N) = K \]  
(21)

2. Second, as long as each individual rationally forecasts the future development of the whole market and new entries by competitors at the optimal entry threshold, we get (Bartolini, 1993; proposition 1; Grenadier, 2002, p. 699): 

\[ V_n(\theta^*(n), n, N) = 0 \]  
(22)

3. Third, on (6) for \( n = N < \bar{N} \), yields (8) and \( B(N, N) = 0 \).

Next, differentiating (21) totally with respect to \( n \) and using (22) we get:

\[ 0 = \frac{dV(\theta^*(n), n, N)}{dn} = V_0(\theta^*(n), n, N) \frac{\partial \theta(n)^*}{\partial n} \] 
(23)

\[ = \left[ \frac{u(n)}{\rho - \alpha} + B(n, N) \beta (\theta(n)^*)^{\beta - 1} \right] \frac{\partial \theta(n)^*}{\partial n} \]

This smooth pasting condition states that either each individual exercises its entry option at the level of \( \theta \) at which the value is tangent to the entry cost, i.e., \( V_0(\theta^*, n, N) = 0 \), or the optimal trigger \( \theta^*(n) \) does not change with \( n \). While the former means that the value function is smooth at entry and the trigger is a continuous function of \( n \), the latter indicates that an individual would benefit from marginally anticipating or delaying its entry decision. In particular if \( V_0(\theta^*, n, N) < 0 \) it means that the value of migrate is expected to increase if \( \theta \) drops. On the contrary if \( V_0(\theta^*, n, N) > 0 \) means that an individual would expect to make losses versus a future drop in \( \theta \). In both situations (23) is satisfied by imposing \( \frac{\partial \theta^*}{\partial n} = 0 \).
Condition (23) splits $[0, N]$ into intervals where one of the following two conditions must hold:

$$\left[ \frac{u(n)}{\rho - \alpha} + B(n, N) \beta (\theta^*)^{\beta - 1} \right] = 0 \quad (24)$$

or

$$\frac{\partial \theta^*(n)}{\partial n} = 0 \quad (25)$$

Since $B(N, N) = 0$ and $\frac{u(N)}{\rho - \alpha} > 0$, then (24) cannot hold at $n = N$. Therefore, it must be (25) that hold at $n = N$ and by (21):

$$\theta^*(N) = (\rho - \alpha) \frac{K}{u(N)} \quad (26)$$

Now, define $n^*$ as the largest $n \leq N$ that satisfies (24). For all $n^* \leq n \leq N$, we have $\frac{\partial \theta^*(n)}{\partial n} = 0$, so that all immigrants in the range $[n^*, N]$ must enter at $\theta^*(N)$. In addition, since for the range $n < n^*$ (24) holds, applying this to the general solution (20), gives as optimal range:

$$\theta^*(n) = \frac{\beta}{\beta - 1} (\rho - \alpha) \left( \frac{K}{u(n)} \right) \quad (27)$$

Finally, the solution $n^* < N$ is obtained by combining (26) and (27), i.e.,

$$\theta^*(n^*) = \theta^*(N) \Rightarrow \frac{\beta}{\beta - 1} (\rho - \alpha) \left( \frac{K}{u(n^*)} \right) = (\rho - \alpha) \frac{K}{u(N)}$$

Let us now demonstrate the uniqueness of $n^*$. First, by $B(N, N) = 0$, at $N$, $V(\theta, N, N)$ equals the discounted income stream with benefit fixed at $u(N)$:

$$V(\theta, N, N) = \hat{V}(\theta, N) \equiv \frac{\theta(t) u(N)}{\rho - \alpha} \quad (28)$$

Then, to obtain $B(n, N)$, substitute (20) into (22): $B_n(n, N) = - (\theta^*)^{1-\beta} u'(n) / (\rho - \alpha)$ and integrating between $n$ and $N$, gives:

$$\int_n^N B_q(q, N) dq = - \int_n^N (\theta^*)^{1-\beta} \frac{u'(q)}{\rho - \alpha} dq \quad (29)$$

Using (1), $B(N, N) = 0$, and changing the integration variable on the right-hand side of (29), gives

$$B(n, N) = \frac{\pi^{1-\beta}}{\beta (\rho - \alpha)} [u^\beta(N) - u^\beta(n)] < 0 \quad (30)$$

with $\lim_{n \to N} B(n, N) = 0^-$. Substituting (30) into (23), we can define the function:

$$H(n) = \frac{u(n)}{\rho - \alpha} + \left( \frac{\pi^*}{\theta^*} \right)^{1-\beta} \frac{1}{(\rho - \alpha)} [u^\beta(N) - u^\beta(n)] \quad (31)$$

with $H(N) > 0$. If $H$ is still positive for a $N - y$ (where $y$ may be infinitesimally small), with $\frac{\pi^*}{\theta^*} = u(N)$ we ought to obtain $\frac{\partial H}{\partial n} = 0$. This procedure continues until we obtain $y^*$ (defined by $n^* = N - y^*$) such that $H(n^*) = 0$. Let us take the first derivative with respect to $y$.
\[
\frac{dH(N-y)}{dy} = -u'(N-y) \frac{\rho}{\rho - \alpha} + \frac{\beta}{\theta} \left( \frac{\beta \theta}{\theta} \right)^{1-\beta} \frac{1}{\theta} \beta u^{\beta-1}(N-y) \ u'(N-y) u(N-y) \ 2
\]

\[
\frac{u'(N-y)}{\rho - \alpha} \left[ \frac{\beta}{\rho - \alpha} \int_0^\infty u^\beta(N) \ g(N) \ dN - u^\beta(n) \right]
\]

\[
\frac{u'(N-y)}{\rho - \alpha} \left[ (u(N))^{1-\beta} \beta u^{\beta-1}(N-y) - 1 \right]
\]

\[
\frac{u'(N-y)}{\rho - \alpha} \left[ \beta \left( \frac{u(N) - u(N)}{u(N)} \right)^{\beta-1} - 1 \right] < 0
\]

Q.E.D. (Quod erat demonstrandum) if \( y \) increases (moving from \( N \) to 0) there exists a value of \( n^* \) (i.e. \( y^* \)) such that \( H(n^*) = 0 \).

### B Proof of Proposition 2.

With uncertainty over stock \( N \), equation (30) becomes:

\[
E(B(n)) = \frac{\pi^{1-\beta}}{\beta (\rho - \alpha)} \left[ \int_0^\infty u^\beta(N) \ g(N) \ dN - u^\beta(n) \right]
\]

(33)

which is negative because it worth \( u^\beta(n) > u^\beta(N) \) for any \( N > n \). Furthermore, the limit of \( E(B(n)) \), yields:

\[
\lim_{n \to N} E(B(n)) = \frac{\pi^{1-\beta}}{\beta (\rho - \alpha)} \left[ \int_0^\infty u^\beta(N) \ f(N) \ dN - u^\beta(n) \right]
\]

\[
= \frac{\pi^{1-\beta}}{\beta (\rho - \alpha)} \left[ \frac{-u^\beta(n) f(n)}{-f(n)} - u^\beta(n) \right]
\]

\[
= \frac{\pi^{1-\beta}}{\beta (\rho - \alpha)} \left[ u^\beta(n) - u^\beta(n) \right] = 0
\]

which is consistent with (11).26

The smooth pasting condition strongly depends on \( E(B(n)) \):

\[
E(H(n)) = \frac{u(n)}{\rho - \alpha} + \beta \theta \ u^{\beta-1} E(B(n))
\]

(34)

\[
= \frac{u(n)}{\rho - \alpha} + \left( \frac{\pi^{1-\beta}}{\theta} \right) \frac{1}{\theta} \beta u^{\beta-1}(N-y) \ u'(N-y) u(N-y)
\]

Since when \( n \to \bar{N} \) we get \( E(B(n)) = 0 \), the smooth pasting reduces to \( E(H(\bar{N})) = \frac{u}{\rho - \alpha} > 0 \) which requires that \( \frac{d\theta}{dN} = 0 \).27

For the uniqueness of \( n^* \), assuming that \( E \left( H (\bar{N} - y) \right) > 0 \) so that \( \frac{dE}{dy} > 0 \) and the optimal trigger is \( \pi^* \), we need to show that \( \frac{dE(H(\bar{N} - y))}{dy} < 0 \).

Substituting \( \bar{N} - y \) into (34), we get:

\[
E(H(\bar{N} - y)) = \frac{u(\bar{N} - y)}{\rho - \alpha} + \left( \frac{u^{1-\beta}}{\rho - \alpha} \right) \left[ \int_{\bar{N} - y}^\infty u^\beta(x) f(x) \ dx \right]
\]

\[
- \frac{1}{\rho - \alpha} \left[ \frac{\beta}{\rho - \alpha} \int_0^{\bar{N} - y} u^\beta(N) \ f(N) \ dN - u^\beta(\bar{N} - y) \right]
\]

\[
\frac{E(H(\bar{N} - y))}{\rho - \alpha} = \frac{u(\bar{N} - y)}{\rho - \alpha} + \left( \frac{u^{1-\beta}}{\rho - \alpha} \right) \left[ \int_{\bar{N} - y}^\infty u^\beta(x) f(x) \ dx \right]
\]

\[
- \frac{1}{\rho - \alpha} \left[ \frac{\beta}{\rho - \alpha} \int_0^{\bar{N} - y} u^\beta(N) \ f(N) \ dN - u^\beta(\bar{N} - y) \right]
\]
Taking the derivative:

\[
d \frac{E(H(\bar{N} - y))}{dy} = - \frac{u'(\bar{N} - y)}{\rho - \alpha} + \left(\frac{\beta}{\rho - \alpha}\right)^{1-\beta} \times \\
\times \left[-u^\beta(\bar{N} - y)f(\bar{N} - y)(1 - F(\bar{N} - y)) \frac{\int_{N-y}^{\infty} u^\beta(x)f(x)dx f(\bar{N} - y)}{(1 - F(\bar{N} - y))^2} + \beta u^{\beta-1}(\bar{N} - y)u'(\bar{N} - y)\right]
\]

\[
= \left[-u^\beta(\bar{N} - y)f(\bar{N} - y) \frac{\int_{N-y}^{\infty} u^\beta(x)f(x)dx f(\bar{N} - y)}{(1 - F(\bar{N} - y))^2} + \beta u^{\beta-1}(\bar{N} - y)u'(\bar{N} - y)\right] + \frac{\beta}{\rho - \alpha} \times \\
\times \left[\left[-u^\beta(\bar{N} - y)f(\bar{N} - y) \frac{\int_{N-y}^{\infty} u^\beta(x)f(x)dx f(\infty - y)}{(1 - F(\bar{N} - y))^2} + \beta u^{\beta-1}(\bar{N} - y)u'(\bar{N} - y)\right] \right]
\]

\[
= \frac{dE(H(\bar{N} - y))}{dy} = \frac{u'(\bar{N} - y)}{\rho - \alpha} \left[-1 + (u^{1-\beta})\beta u^{\beta-1}(\bar{N} - y)\right] + \\
\left[\left[-u^\beta(\bar{N} - y)f(\bar{N} - y) \frac{\int_{N-y}^{\infty} u^\beta(x)f(x)dx f(\infty - y)}{(1 - F(\bar{N} - y))^2} + \beta u^{\beta-1}(\bar{N} - y)u'(\bar{N} - y)\right] \right] (35)
\]

Then, there exists a value \( n^* = \bar{N} - y^* \) such that \( E(H(n^*)) = 0 \).

Finally we need to show that \( n^* < n^* \). To do this we need to show two conditions:

1. The value of \( H(N - y) \) is greater than the value of \( E[H(N - y)] \) for any \( y > y^* \).

2. The function (34) increases more rapidly than (31), i.e., the derivative (32) is greater than (35).

Condition 2 combined with condition 1 implies that the two functions do not intersect and that there exists a \( y^* \) such that \( E[H(N - y^*)] = 0 \). For the first condition, stressing the analysis with respect to any point \( y \) greater than \( y^* \), we can show that (34) evaluated at \( N - y \) (i.e., assuming \( N \) as the upper limit of the stock) is lower than (31) if and only if:

\[
\frac{\int_{N-y}^{\infty} u^\beta(x)f(x)dx}{1 - F(N - y)} < 0
\]

that follows using the neoclassical properties. For the second condition, comparing (32) with (35) evaluated at \( N - y \), we can show that:

\[
\frac{dH(N - y)}{dy} > \frac{dE(H(N - y))}{dy}
\]

This result can be shown in the following figure 4:
Figure 4: Graphic Solution
A "quota" is defined as the share of a total immigrants that is assigned to a particular group. In this paper the term "quota" is in line with SOPEMI International Migration Outlook (2006) and it is foregone in favour of terms that more precisely describe the nature of numerical migration level, such as "target level", "numerical limit", the maximum and "cap".

Schäuble-Sarkozy suggested that EU asylum policy should be centralised, that long-term economic immigration should be managed by quotas and that short-term immigration should be regulated by temporary visas", Editorial of Intereconomics, (2006).

In their broadest sense, regularization programs offer those migrants who are in a country without authorization the opportunity to legalize their status.

Irregular migrants, also referred to as "undocumented," "unauthorized," or "illegal," are defined by most states as those migrants who have either entered a country legally and then fallen out of legal status — such as students, temporary workers, rejected asylum seekers, or tourists — or those who have entered illegally, either by crossing a border undetected or with false documents. In either case, irregular migrants do not have a legal right to residence in the state to which they have migrated.

Bartolini (1993, 1995), develops a general model that considers the investment decision of decentralized profit-maximizing agents, who face investment adjustment costs in a market with stochastic returns and a limit on aggregate investment. The model is consistent with equilibrium models of asset pricing under uncertainty but differs from the mainstream assumption of constant investment cost assuming that, for technological or institutional reason, the investment cost is constant only until an investment ceiling becomes binding. At that point, in fact, Bartolini shows that cost becomes infinite. His paper shows that a competitive market reacts to this type of externality by generating recurrent runs as aggregate investment approaches its limit.

The existence of quotas seems to be idiosyncratic with respect to various aspects of the economic approach. Particularly, it can be used not only to migration phenomenon, by also concerning foreign investment or also the adoption of licenses regulating the market. We can find many examples in which quotas assume an important role in the market. Capital controls are often imposed to prevent a country’s net credit position from exceeding some acceptable levels; central banks face limits on the amount of foreign reserves that can be used to enforce an exchange rate target; firms in a fast-growing industry or in a developing economy may be competing for extended periods for a small number of qualified managers or highly skilled workers; entry of firms is restricted in many industries by regulations aimed at containing market size or by technological constraints on the use of a scarce resource. Similar approaches arise for taxi and liquor licences, fishing and coastal trade rights, the number of polluting trade permits or ecolabelling permits (Dosi and Moretto, 2001).

The indexes from 1 to 6 were defined by Fondazione Rodolfo Debenedetti (see www.frdh.org for details) and the index 7 was defined by Hatton (2004).

"All countries except Greece, [...] denote a tightening in regulations", see Boeri and Brücker, 2005, page 634.

Table 1 is our elaboration by using some European immigration databases and sources. Table 1 concerns Europe in geographical sense.

Table 2 is our elaboration on Jachimowicz et al. (2004, pages 36-40 ) and Sunderhaus, (2007).

As stressed by Epstein and Nitzan (2006), "empirical evidence from the EU countries shows that immigration had at most a very small impact on wages and employment opportunities of natives". Moreover, "most of the evidence on the effect of immigrants on wages (and employment) for the US is also ambiguous in the sense that some studies show small positive effect and others small negative effects". In line with this empirical evidence we study the migration process without taking into account the crossed effect on natives' wages and unemployment level.

Concerning this, see Bauer, Epstein and Gang, (2002), Epstein and Gang (2004), Moretto
In other words, the reserve value $u$ measures the level of "desperation" by the potential immigrants.

For details about the process, see Dixit and Pindyck (1994, pag. 71).

In this case we assume that the shock is homogeneous for all immigrants. If the shock were individual-specific, the model should change by considering the immigrants as they had different skills (i.e., they could perceive different wage gaps). The result will be a change of scale in the trigger levels and a self-selection of immigrants, but the theoretical result will not change. For more details, see Vergalli (2007), page 12. Therefore, we use a homogeneous shock as a general model.

That is, the sum of the instantaneous dividend (benefit) flow and the expected capital gain equals normal profits (Dixit and Pindyck, 1994, p. 185).

Where $V_0 = \frac{\partial V}{\partial \theta}$ and $V_{\theta\theta} = \frac{\partial^2 V}{\partial \theta^2}$.

That is, the discounted present value of the benefit flows over an infinite horizon starting from $\theta$ (Harrison 1985, p. 44). See equation (19) in the Appendix.

This condition is familiar in the real option theory with the name of matching value condition (see Dixit and Pindyck, 1994).

See condition (23) in the Appendix.

It is worth noting that the "utility" threshold that triggers migration for individual immigrants is identical to that of the individual that correctly anticipates the other immigrants' strategies. This property, discovered first by Leahy (1993), has an important operative implication; i.e., the optimal migration policy of each individual need not take account of the effect of rivals' entry. He/she can behave competitively as if he/she is the last to enter. In other words, when an individual decides to enter, by pretending to be the last to migrate, he/she is ignoring two things: 1) He/she is thinking that his/her benefit flow is given by $u(n)\theta$, with $n$ held fixed forever. Thus, as $u'(n) < 0$, he/she is ignoring that future entry by other members, in response to a higher value of $\theta$, will reduce "utility". All things being equal, this would make entry more attractive for the migrant that behaves myopically. 2) He/she is unaware that the prospect of future entry by competitors reduces the option value of waiting. That is, pretending to be the last to migrate, the individual also believes he/she still has a valuable option of waiting before making an irreversible decision. All things being equal, this makes the decision to enter less attractive. The two effects offset each other, allowing the migrant to act as if in isolation (see Dixit and Pindyck, 1994, p. 291).

Obviously, a government sets the quota in line with the supply-demand gap of the labour market. Indeed, "the selection of candidates for immigration can be made by the receiving country itself [...] In this case, potential immigrants are screened on the basis of certain characteristics, deemed to contribute to, and facilitate, integration in the host country, such as [...] having an occupation deemed to be in shortage and having a prior job offer from an employer in the host country" (Sopemi, 2006, page 114).

The upper support of the distribution can be set to $\bar{N} \leq N$. Without losing in generality we assume that $N = \bar{N}$.

This due to the fact that $\frac{d \bar{u}}{dn} > 0$ and $\frac{du(n)}{dn} < 0$.

We concentrate the analysis on the case of an unknown quota. Obviously, we have the same effect with known quota.

Let us notice that $\lim_{n \to \infty} E(B(n)) = 0$ even if $u(n) \to \bar{u} \geq 0$.

Notice that this result always holds for $\bar{u} > 0$, but it is also true for $\bar{u} \geq 0$ by using limit definition. In this case for each real number $\varepsilon > 0$ infinitesimely small, there exists a value $n'$ such that for $n > n'$ the difference

$E(H(n')) - E(H(\infty)) < \varepsilon$

Nevertheless, since now $E(H(\infty)) = 0$, we are able to find a value $n'$ such that:

$E(H(n')) - 0 < \varepsilon$

If $\varepsilon \to 0$ it follows that $n'$ is the right value we are searching.
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